MATHEMATICS

CANADA

MODULE 5
RATIO AND PROPORTION

Alberta
EDUCATION

Distance
Learning

Mathematics 8

Module 5: Ratio and Proportion

MODULE BOOKLET

Mathematics 8
Student Module
Module 5
Ratio and Proportion

Welcome to Module 5!

*We hope you'll enjoy your study of **Ratio and Proportion**.*

To make your learning a bit easier, a teacher will help guide you through the materials.

So whenever you see this icon,
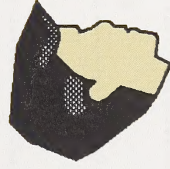
turn on your audiocassette and listen.

CONTENTS AT A GLANCE

What Lies Ahead

In the Module Introduction you will preview the module and learn how the module will be evaluated.

Working Together

In this module you will learn about ratio and proportion.

Part One: Ratios

Part Two: Rates

Part Three: Percents
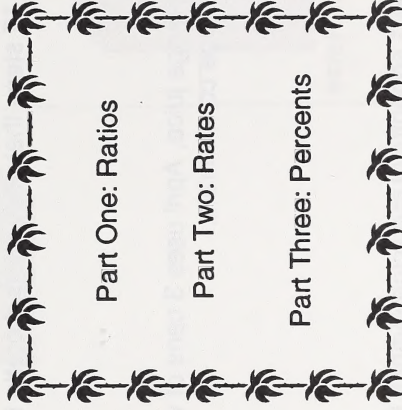
Ratios, rates, and percents are common in everyday life.

Examples of Ratios

- The band class has a ratio of 16 boys to 12 girls.

- For every 12 steps that Jody takes, Loretta takes 9 steps.

- To make orange juice, April uses 3 cans of water for 1 can of orange concentrate.

Examples of Rates

- Harry takes 1 spoonful of medicine every 6 hours.

- Tim drove at a rate of speed of 95 km/h on his recent trip to Calgary.

- The sale price is 12 cans for $4.99.

Examples of Percents

- On Saturday there is a 50% discount on all men's shoes.

- Rania scored 83% on her final math exam.

- The real estate agency makes 7% commission on the sale of a house.

Course Overview
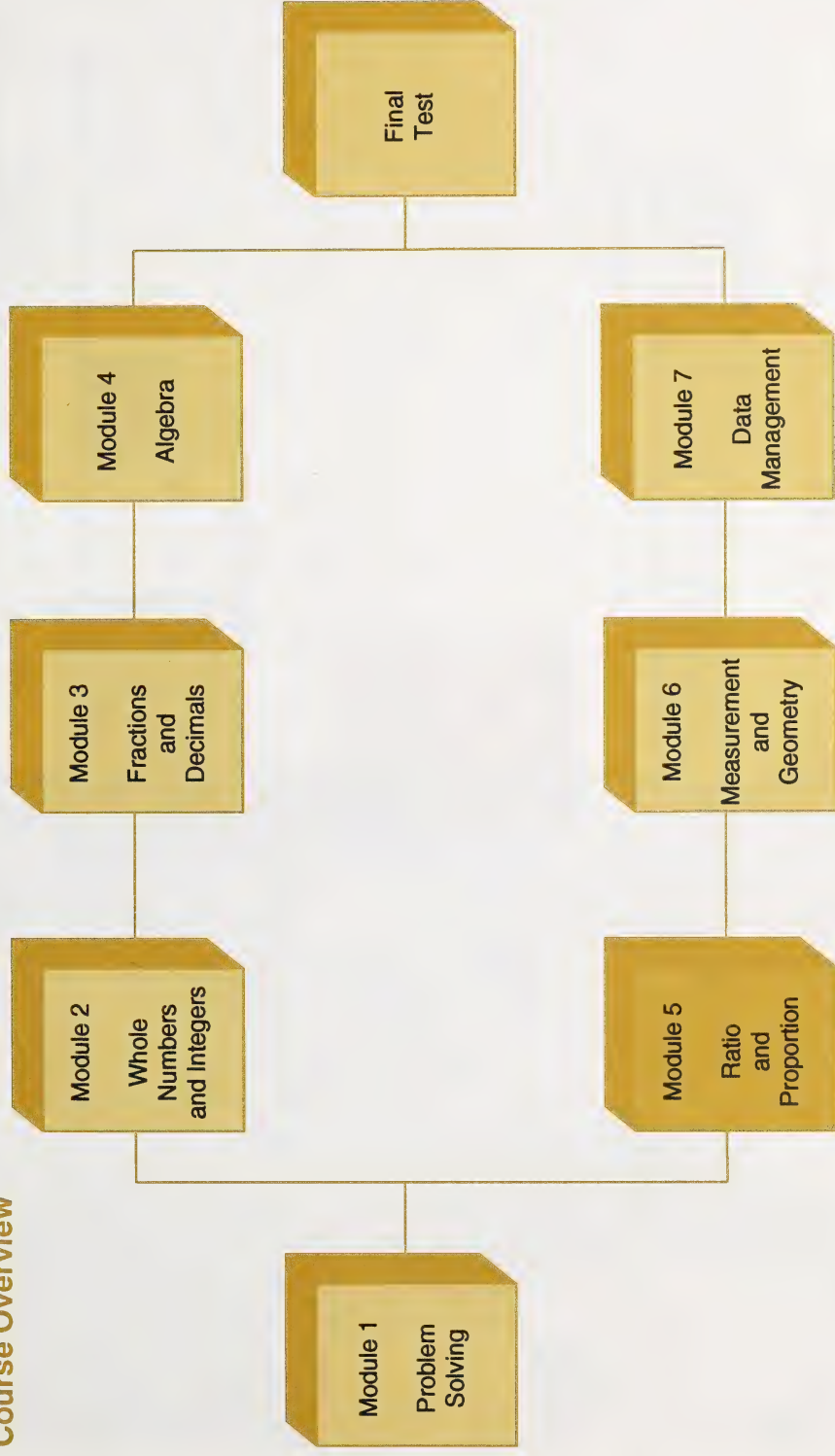
Mathematics 8 has seven modules and a final supervised test. This module booklet is part of Module 5.

Module 1 Components

(optional)

Mathematics 8
Module 5
Ratio and Proportion

Assignment
Booklet
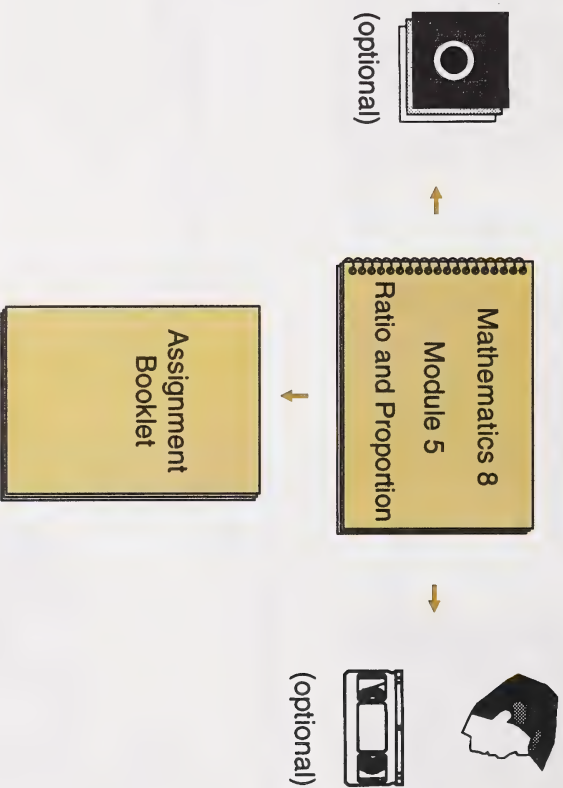
(optional)

This module booklet will give you instruction and practice in learning mathematical skills and words. It will also direct you to the other components of the module. The computer and video activities in this booklet are optional; there are print alternatives. You should see your learning facilitator to check your answers to the activities in this booklet. This module booklet is not to be submitted for a grade.

Your mark on this module will be determined by your work in the Assignment Booklet.

Take time to preview this module booklet before beginning Section 1.

Ratios are used often in the everyday world. You probably already know some things about ratios. This part of the module will build on this knowledge.

Here are some common examples of ratios. What does each of these ratios mean?

- the win-loss ratio

- the pupil-teacher ratio

- the boy-girl ratio

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GETTING SET

What Lies Ahead

This section will test these skills.

- writing a ratio using the colon form, fraction form, and decimal number form

- writing equivalent ratios

- writing ratios in simplest form

- comparing and ordering ratios

- solving ratio problems using proportions

Working Together

Sections 1 to 5 deal with ratios.

The pretest in this section will help you and your learning facilitator determine your strengths and weaknesses.

Pretest

Space for Your Work

1.

a. Write the ratio of the number of nickels to the number of pennies. Use the colon form.

b. Write the ratio of the value in cents of the nickels to the value in cents of the pennies. Use the fraction form.

c. Write the ratio of the value in cents of the nickels to the total value in cents. Use the decimal number form.

2. Write three equivalent ratios for each of the following.

a. 2 out of 3 people voted for Edith Jones.

b. At the County Fair, 20 student tickets were sold for every 5 adult tickets sold.

3. A baseball team won 36 out of 42 games during the season. It won 8 out of 11 games during the playoffs. Was the ratio of the number of games won to the number of games played better during the season or during the playoffs?

4. Matt sank 20 basketballs in 36 shots. Jon sank 10 basketballs in 18 shots. Are these ratios proportional?

Space for Your Work

Space for Your Work

5.   In Tara's class the ratio of the number of girls to the
      number of boys is 3 to 4. There are 12 girls in the
      class. How many boys are there in the class?

6.   The ratio of the measure of uncooked macaroni to
      the measure of cooked macaroni is 1 to 3. How
      much cooked macaroni is made from 350 mL of
      uncooked macaroni?

See your learning facilitator to check
your answers and to receive further
instructions.

Notes

Notes

READING AND WRITING RATIOS
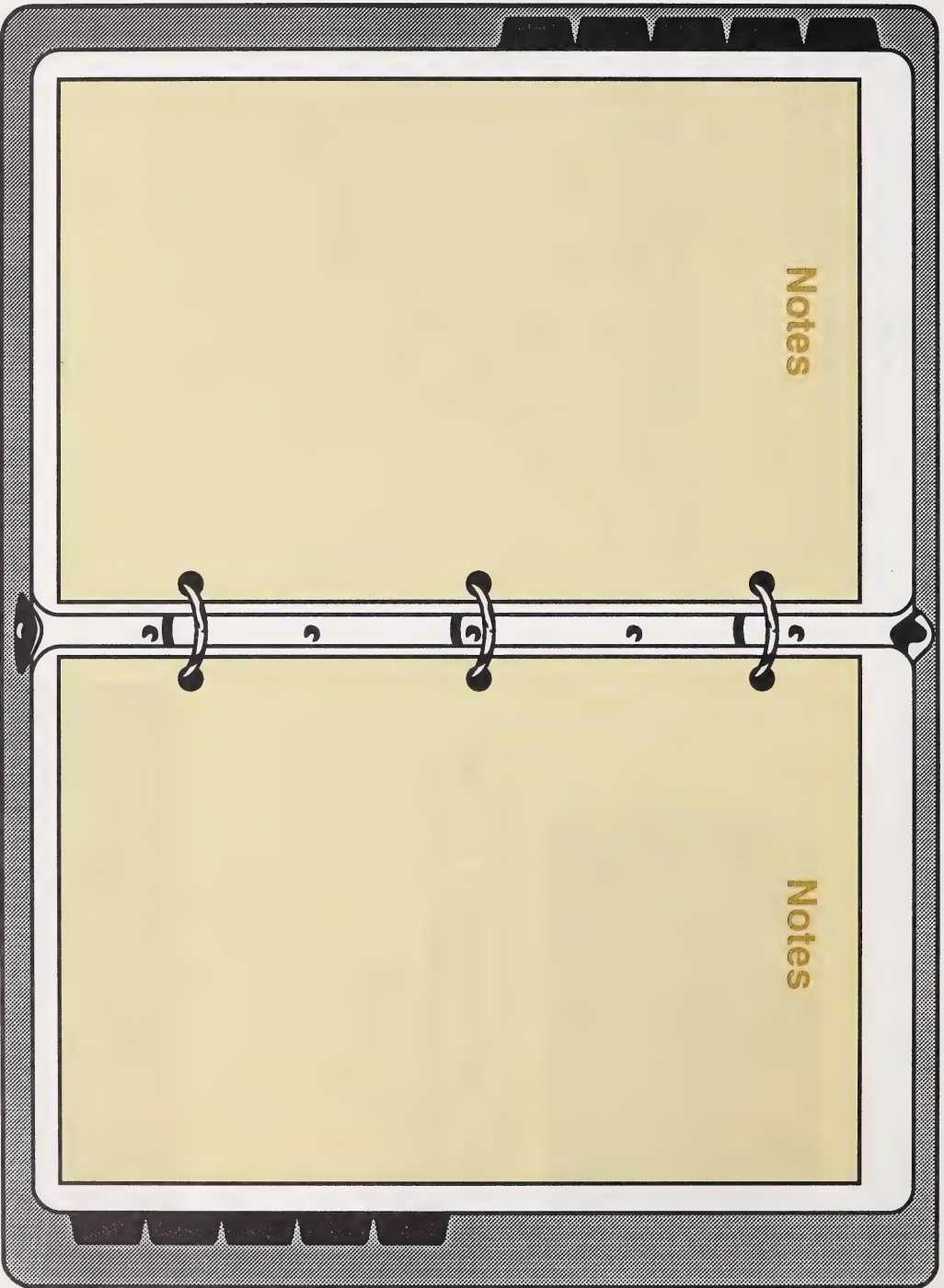
What Lies Ahead

In this section you will review these skills.

* reading a ratio

* interpreting a ratio

* writing a ratio using a colon

* writing a ratio using its fraction form

* writing a ratio using its decimal number form

In this section you will review the meaning of these words.

* ratio

* term

* first term, second term, third term

* two-term ratio

* three-term ratio

Working Together

In the Introductory Activities you will work with pattern blocks to review the meaning of **ratios**. Then you will review how to read a ratio and how to write a two-term ratio in the colon form, the fraction form, and the decimal number form. Finally, you will explore three-term ratios.

Introductory Activities

For Questions 1 to 3 you will require hexagons, triangles, and parallelograms. These shapes can be cut out of the Appendix at the back of the book.

Arrange the shapes in the patterns shown in Figure 1 and Figure 2.
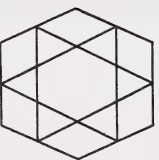
Figure 1   Figure 2
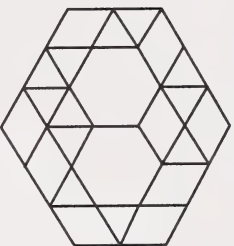
1. Compare the number of hexagons in Figure 1 to the number of parallelograms in Figure 1.

a. The number of hexagons in Figure 1 is ▧ .

b. The number of parallelograms in Figure 1 is ▧ .

c. The ratio of the number of hexagons in Figure 1 to the number of parallelograms in Figure 1 is ▧ to ▧ .

2. Compare the number of triangles in Figure 1 to the number of parallelograms in Figure 2.

   a. The number of triangles in Figure 1 is ▨ .

   b. The number of parallelograms in Figure 2 is ▨ .

   c. The ratio of the number of triangles in Figure 1 to the number of parallelograms in Figure 2 is ▨ to ▨ .

3. Compare the number of triangles in Figure 2 to the total number of triangles used for both figures.

   a. The number of triangles in Figure 2 is ▨ .

   b. The total number of triangles for both figures is ▨ .

   c. The ratio of the number of triangles in Figure 2 to the total number of triangles for both figures is ▨ to ▨ .

See your learning facilitator to check your answers and to receive further instructions.

Working Together

As you learned from the Introductory Activities, there are many different **two-term ratios** that can be used to describe a situation.

Example 1

An aquarium contains 3 orange fish and 5 black fish. Write statements comparing the number of fish in the aquarium.

Solution

- The ratio of the number of orange fish to the number of black fish is 3 to 5.

- The ratio of the number of black fish to the number of orange fish is 5 to 3.

- The ratio of the number of orange fish to the total number of fish is 3 to 8.

- The ratio of the number of black fish to the total number of fish is 5 to 8.

Note

When you use ratios, be sure to write a statement that indicates what is being compared. Also make sure that the order of the numbers in the ratio matches the written order of the items being compared.

Example 2

A large tube of toothpaste contains 200 mL. A small tube contains 100 mL. Write statements comparing these quantities.

Solution

- The ratio of the amount of toothpaste in the small tube to the amount of toothpaste in the large tube is 100 to 200.

- The ratio of the amount of toothpaste in the large tube to the amount of toothpaste in the small tube is 200 to 100.

- The ratio of the amount of toothpaste in the small tube to the total amount of toothpaste is 100 to 300.

- The ratio of the amount of toothpaste in the large tube to the total amount of toothpaste is 200 to 300.

Note

When you are comparing quantities with the same units, the units are not mentioned.

Writing Ratios with Colons

A colon can also be used to express a ratio.

Example 1

There are 5 pencils and 7 brushes in a container. Write the ratio of the number of pencils to the number of brushes. Use the colon form.
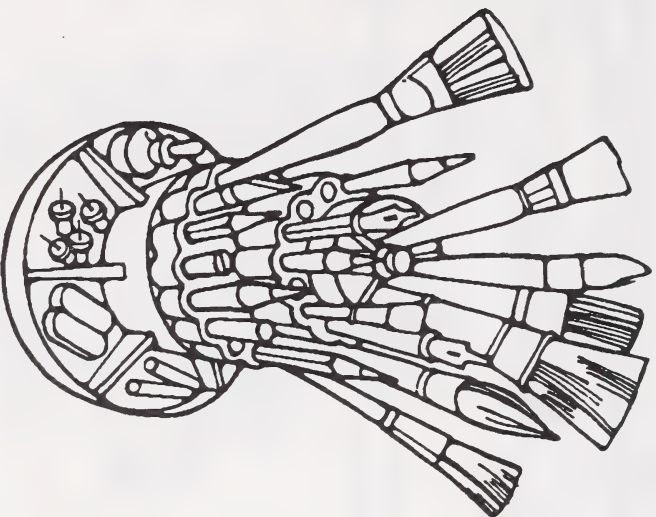
Solution

The ratio of the number of pencils to the number of brushes is 5 to 7.

When written in colon form, the ratio looks like this.

5 : 7

This is read as "5 to 7."

Note

5 is the **first term** of the ratio, and 7 is the **second term** of the ratio.

Example 2

Angelica saw 5 squirrels and 7 rabbits at the park. Write the ratio of rabbits to squirrels. Use the colon form.

Solution

The ratio of rabbits to squirrels is 7 to 5.

When written in colon form, the ratio looks like this.

$$7 : 5$$

This is read as "7 to 5."

Note

7 is the first term of the ratio, and 5 is the second term of the ratio.

Writing Ratios in Fraction Form

A ratio can also be expressed in fraction form.

Example 1

An office employs 2 women and 1 man. Write the female-male ratio in fraction form.

Solution

The female-male ratio is 2 to 1.

When written in fraction form, the ratio looks like this.

$$\frac{2}{1}$$

This is read as "2 to 1."

You can also say that the number of females is twice the number of males.

Note

2 is the first term of the ratio, and 1 is the second term of the ratio.

Example 2

There are 5 soccer balls and 1 football. Write the ratio of the number of footballs to the total number of balls. Use the fraction form.

Solution

The ratio of the number of footballs to the total number of balls is 1 to 6.

The ratio looks like this when written in fraction form.

$$\frac{1}{6}$$

This is read as "1 to 6."

You can say that the number of footballs is one-sixth of the total number of balls.

Note

1 is the first term of the ratio, and 6 is the second term of the ratio.

Writing Ratios in Decimal Number Form

Occasionally, ratios are written in their decimal number form.

Example 1

The **gear ratio** of a bicycle compares the number of teeth on the front gear to the number of teeth on the back gear. What is the gear ratio of a ten-speed bicycle in which the front gear has 52 teeth and the back gear has 28 teeth? Use the decimal number form.
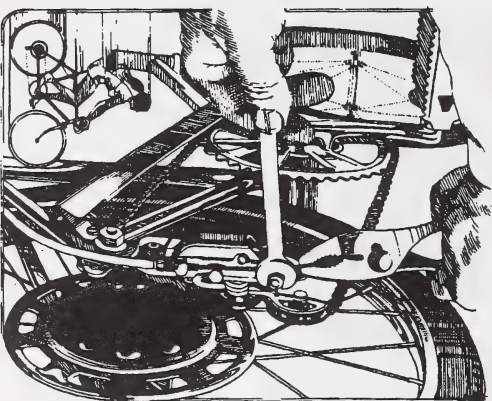
Solution

To get the decimal number form, first write the ratio as a fraction.

$$\frac{52}{28}$$

Then use division to change the fraction into a decimal number.

$$\frac{52}{28} \doteq 1.85 \quad \longleftarrow$$

This is read as "1 point 85."

The gear ratio is about 1.85.

Example 2

Alanna bought 6 cans of orange pop and 5 cans of cola.
What is the ratio of cans of orange pop to cans of cola?
Use the decimal number form.
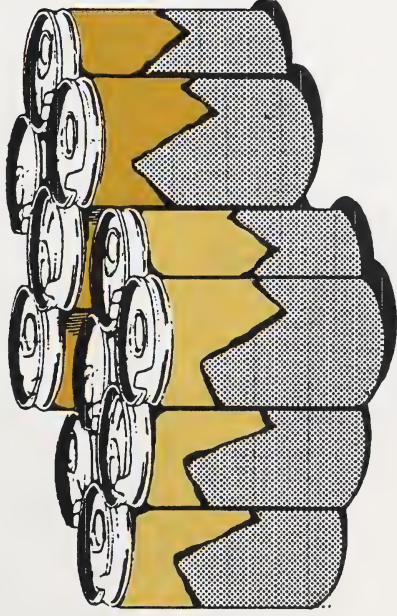
Solution

First write the ratio as a fraction.

$$\frac{6}{5}$$

Then change to a decimal number by dividing.

$$\frac{6}{5} = 1.2 \longrightarrow \text{This is read as "1 point 2."}$$

The ratio of cans of cola to cans of orange soda is 1.2.

Practice Activities

Print Alternative

1. Write statements comparing the capacities of these spice bottles. Use the ratios that follow.

   Oregano 28g

   Dill Weed 13g

   a. 28 to 13
   b. 13 to 41
   c. 28 to 41

2. There are 6 balls in a group of 10 items of sports equipment. What is the ratio of the number of balls to the total number of equipment items? Use the fraction form.

Space for Your Work

3. There are 5 roses and 8 daisies in a bouquet of flowers. Use the fraction form to write the following ratios.

   a. the number of roses to the number of daisies

   b. the number of roses to the total number of flowers

4. The Hawks hockey team won 41 games, lost 10 games, and tied 29 games. Use the colon form to write the win-loss ratio.
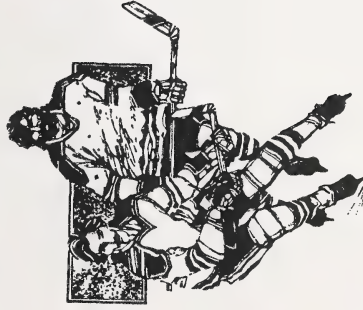
5. Buns come in packages of 8. Wieners come in packages of 12. Use the colon form to write the ratio of the number of wieners to the number of buns.

Space for Your Work

6. A five-speed bicycle has the gear ratios shown in the table at the right. Complete the table by expressing the gear ratios in decimal number form rounded to the nearest thousandth.

6. a.

Teeth on the Front Gear	Teeth on the Back Gear	Gear Ratio
40	28	
40	24	
40	20	
40	17	
40	14	
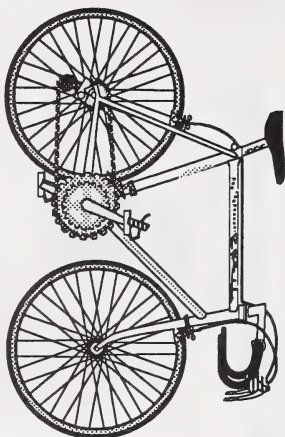
7. There are 128 students and 5 teachers in a school. Write the student-teacher ratio. Use the decimal number form rounded to the nearest tenth.

Computer Alternative

Space for Your Work
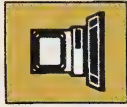
8. Do Lessons 1 to 5 of the disk *Ratio and Percent* from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA).

   Read the instructions in the folder with the disk before using the program. Remember, if you need help, press the SHIFT key and the ? key.

See your learning facilitator to check your answers and to receive further instructions.

Working Together

So far, all of the ratios you have worked with have had two terms. You will now learn about ratios that have more than two terms.

Example

A bag of potting soil contains 5 parts of loam, 2 parts of peat moss, and 1 part of coarse sand. Different **three-term ratios** can describe this situation.

- The ratio of loam to peat moss to coarse sand is 5 to 2 to 1.

- The ratio of loam to coarse sand to peat moss is 5 to 1 to 2.

- The ratio of peat moss to coarse sand to loam is 2 to 1 to 5.

- The ratio of peat moss to loam to coarse sand is 2 to 5 to 1.

- The ratio of coarse sand to peat moss to loam is 1 to 2 to 5.

- The ratio of coarse sand to loam to peat moss is 1 to 5 to 2.

The number of parts of potting soil is 5 + 2 + 1 = 8. So, different two-term ratios can be written as well.

- The ratio of loam to potting soil is 5 to 8.

- The ratio of peat moss to potting soil is 2 to 8.

- The ratio of coarse sand to potting soil is 1 to 8.

Three-term ratios can be written using words or the colon form.

Two-term ratios can be written using words, the colon form, the fraction form, or the decimal number form.

Concluding Activities

1. Jill is mixing different colours of paint to create a shade of brown. To make this shade, she mixes 8 parts of red paint, 5 parts of yellow paint, and 1 part of black paint.
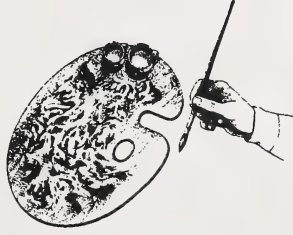
a. What is the ratio of the quantity of red to yellow to black?

b. What is the ratio of the quantity of black to red to yellow?

c. What is the ratio of the quantity of yellow to red to black?

d. What is the ratio of the quantity of red to brown?

e. What is the ratio of the quantity of yellow to brown?

f. What is the ratio of the quantity of black to brown?

2. Sam is 12 years old, Sandra is 13 years old, and Stella is 15 years old.

a. What is the ratio of Sam's age to Sandra's age to Stella's age?

b. What is the ratio of Stella's age to Sandra's age to Sam's age?

c. What is the ratio of Sam's age to the total age of all three children?

d. What is the ratio of the sum of Sandra's age and Sam's age to the total age of all three children?

e. What is the ratio of Stella's age to the total age of all three children?

See your learning facilitator to check your answers and to receive further instructions.

FINDING EQUIVALENT RATIOS

Working Together

In Module 3 you learned how to write equivalent fractions.

This section will show you how to write **equivalent ratios.**

What Lies Ahead

In this section you will learn these skills.

* finding equivalent ratios

* writing ratios in lowest terms

In this section you will learn these terms.

* equivalent ratios

* lowest-term ratio

You can use multiplication or division to write equivalent ratios.

Example 1

The ratio of the number of triangles to the number of circles is 2 to 3.

△△
○○○

Use multiplication to write two equivalent ratios.

Solution

- You can multiply both terms by 2 to find an equivalent ratio.

△△ △△
○○○ ○○○

$$\frac{2}{3} \overset{\times 2}{\underset{\times 2}{=}} \frac{4}{6}$$
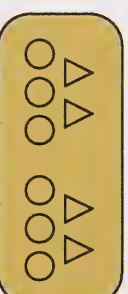
4 to 6 is an equivalent ratio of 2 to 3.

- You can multiply both terms by 4 to find another equivalent ratio.

△△ △△ △△ △△
○○○ ○○○ ○○○ ○○○

$$\frac{2}{3} \overset{\times 4}{\underset{\times 4}{=}} \frac{8}{12}$$
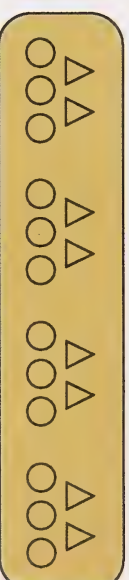
8 to 12 is another equivalent ratio of 2 to 3.

Example 2

The ratio of the number of jacks to the number of balls is 8 to 4.
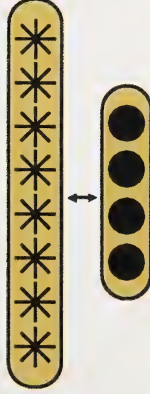
Use division to write two equivalent ratios.

Solution

- To find an equivalent ratio, you can divide both terms by 2.

$$\overset{\div 2}{\overbrace{\frac{8}{4}}} = \underset{\div 2}{\underbrace{\frac{4}{2}}}$$

4 to 2 is an equivalent ratio of 8 to 4.

- To find another equivalent ratio, you can divide both terms by 4.

$$\overset{\div 4}{\overbrace{\frac{8}{4}}} = \underset{\div 4}{\underbrace{\frac{2}{1}}}$$
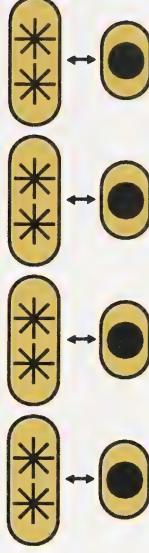
2 to 1 is another equivalent ratio of 8 to 4.

The ratio 2 to 1 is called a **lowest-term ratio**. This means that the terms in the ratio are the lowest whole numbers possible.

Introductory Activities

1. Use diagrams and multiplication to write an equivalent ratio for each of the following ratios.

   a. The ratio of the number of boxes to the number of bags is 3 to 5.

   b. The ratio of the number of squares to the number of circles is 7 to 5.

Space for Your Work

2. Use diagrams and division to express each of the following ratios in lowest terms.

    a. The ratio of the number of cars to the number of bicycles is 8 to 12.

    b. The ratio of the number of dogs to the number of cats is 5 to 10.

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Sometimes the terms of a ratio contain fractions or decimals.

Example

In a punch recipe there are 2.7 L of pineapple juice and 1.3 L of orange juice. What is the ratio of orange juice to pineapple juice? Express the ratio in lowest terms.
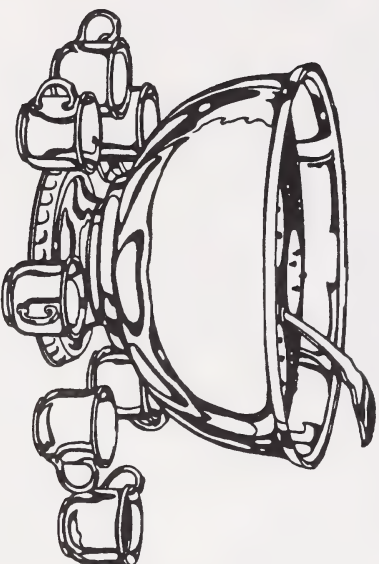
Solution

Write the ratio of orange juice to pineapple juice.

$$1.3 : 2.7$$

To express the ratio in lowest terms, multiply each term by 10. This is necessary because the lowest terms must be whole numbers.

$$\frac{1.3}{2.7} \overset{\times 10}{\underset{\times 10}{=}} \frac{13}{27}$$

The ratio of orange juice to pineapple juice is 13 to 27.

Example 2

Jason painted the fence for $1\frac{3}{4}$ h. Ruth painted the fence for $1\frac{1}{2}$ h. What is the ratio of the time Jason spent to the time Ruth spent? Express the ratio in lowest terms.

Solution

Write the ratio of the time Jason spent to the time Ruth spent.

$$1\frac{3}{4} : 1\frac{1}{2}$$

To express the ratio in lowest terms, first express the terms as improper fractions.

$$\frac{7}{4} : \frac{3}{2}$$

Next express the terms as equivalent fractions with common denominators.

$$\frac{7}{4} : \frac{3}{2} = \frac{7}{4} : \frac{6}{4}$$

Now multiply by 4 to simplify.

$$4 \times \frac{7}{4} : \frac{6}{4} \times 4$$

$$= 7 : 6$$

The ratio of the time Jason spent to the time Ruth spent is 7 to 6.

Practice Activities

Space for Your Work

1. The body of an adult contains about 4.7 L of blood. A blood donor usually gives 0.470 L of blood. What is the ratio of the quantity of the donated blood to the quantity of blood in the body? Express the ratio in lowest terms.

2. In one day a man inhales 11.4 cm³ of air. About 0.57 cm³ of oxygen in this air is absorbed into the bloodstream. What is the ratio of the volume of air inhaled to the volume of oxygen absorbed? Express the ratio in lowest terms.

3. A Minute Beetle is 0.02 cm long. A Goliath Beetle is 14.86 cm long. What is the ratio of the length of the Minute Beetle to the length of the Goliath Beetle? Express the ratio in lowest terms.

4. Jackson slept for $6\frac{1}{2}$ hours on Monday night and $7\frac{1}{3}$ hours on Tuesday night. What is the ratio of the time he slept on Monday night to the time he slept on Tuesday night? Express the ratio in lowest terms.

5. The largest blue diamond in the world is the Hope Diamond which weighs $44\frac{2}{5}$ carats. Another famous diamond, the Timkin Stone, weighs $28\frac{3}{4}$ carats. What is the ratio of the weight of the Hope Diamond to the weight of the Timkin Stone? Express the ratio in lowest terms.
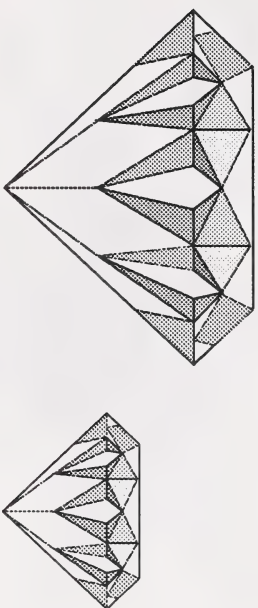
Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

![icon] **Working Together**

In Section 2 you learned about three-term ratios.

Three-term ratios can also be written in simplest form.

Example

Joe's father is 175 cm tall, his mother is 160 cm tall, and he is 200 cm tall. What is the ratio of Joe's height to his father's height to his mother's height? Express the ratio in lowest terms.
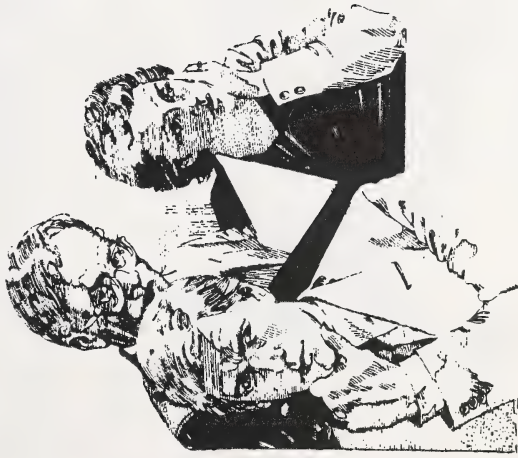
Solution

Write the ratio of Joe's height to his father's height to his mother's height.

200 to 175 to 160

To find the lowest terms of the ratio, you must divide all the terms of the ratio by their greatest common factor.

> The greatest common factor of 200, 175, and 160 is 5. So, divide by 5.

200 : 175 : 160 = 40 : 35 : 32

The lowest-term ratio of Joe's height to his father's height to his mother's height is 40 to 35 to 32.

Concluding Activities

1. At the 1988 Winter Olympics in Calgary, West Germany won 2 gold medals, 4 silver medals, and 2 bronze medals. Switzerland won 5 gold medals, 5 silver medals, and 5 bronze medals.

   a. What is the ratio of the number of gold medals to silver medals to bronze medals won by West Germany? Express the ratio in lowest terms.

   b. What is the ratio of the number of gold medals to silver medals to bronze medals won by Switzerland? Express the ratio in lowest terms.

   c. What do you think is a disadvantage of expressing ratios in lowest terms?

Space for Your Work

2. To make a cement foundation, 300 kg of gravel are mixed with 450 kg of sand and 100 kg of cement. What is the ratio of gravel to sand to cement? Express the ratio in lowest terms.
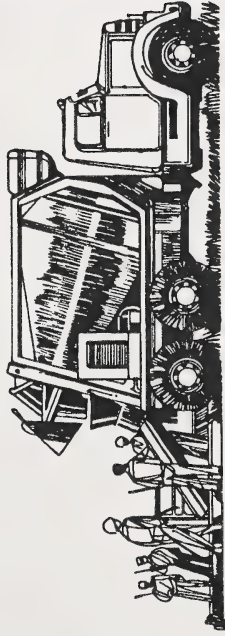
3. During a hockey season a team had 36 wins, 24 losses, and 8 ties. What is the ratio of wins to losses to ties? Express the ratio in lowest terms.
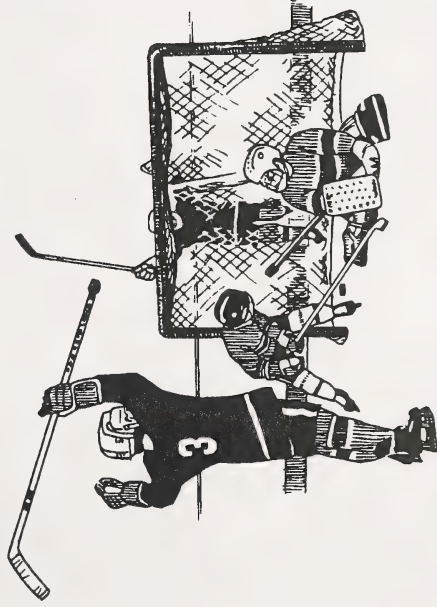
4. Mars is $\frac{1}{7}$ of the earth's size. Mercury is $\frac{1}{16}$ of the earth's size. What is the ratio of the size of Earth to the size of Mars to the size of Mercury? Express the ratio in lowest terms.

5. Saturn is 730 times the size of Venus. Neptune is about $\frac{1}{12}$ of the size of Saturn. What is the ratio of the size of Venus to the size of Saturn to the size of Neptune? Express the ratio in lowest terms.

See your learning facilitator to check your answers and to receive further instructions.

Space for Your Work

COMPARING AND ORDERING RATIOS

What Lies Ahead

In this section you will learn these skills.

- comparing ratios

- ordering ratios from largest to smallest or from smallest to largest

- determining if situations are proportional

In this section you will learn these terms.

- golden ratio

- proportional ratio

Working Together

Comparing and ordering are important skills in mathematics.

In Module 2 you compared and ordered whole numbers and integers.

$$832 \; > \; 821$$

$$-5 \; > \; -8$$

In Module 3 you compared and ordered fractions and decimal numbers.

$$\frac{3}{4} \; > \; \frac{1}{4}$$

$$\frac{5}{8} \; > \; \frac{1}{2}$$

$$0.75 \; > \; 0.25$$

$$0.8 \; > \; 0.15$$

In this section you will compare and order ratios.

Example 1

In a cake recipe, the ratio of the measure of flour to the measure of sugar is 3 : 1. In another cake recipe, the ratio of the measure of sugar to the measure of flour is 3 : 1. Which recipe makes sweeter cakes?
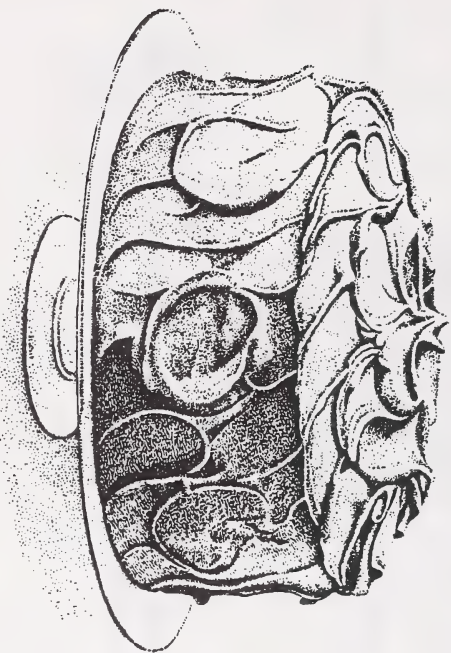
Solution

The recipe with the greater ratio of sugar to flour will be sweeter.

The ratios in the problem are not written in the same order. Rewrite the ratios so that they are comparing the amount of sugar to the amount of flour.

$$\frac{sugar}{flour}$$

	Recipe 1	Recipe 2
	$\frac{1}{3}$	$\frac{3}{1}$

Now compare the ratios.

$$\frac{1}{3} < \frac{3}{1}$$

So, the second recipe is sweeter.

Example 2

Harry hit safely 5 out of 9 times at bat. Willy hit safely 7 out of 9 times at bat. Which batter has the greater ratio of hits to times at bat?

Solution

Because Harry and Willy have both been up to bat 9 times, it is easy to compare the ratios.

Write the ratios of hits to times at bat.

	Willy	Harry
	$\dfrac{7}{9}$	$\dfrac{5}{9}$

$\dfrac{number\ of\ hits}{number\ of\ times\ at\ bat}$

Because the second terms are the same, you can compare the first terms.

$$7 > 5$$

So, $\dfrac{7}{9} > \dfrac{5}{9}$.

Willy has the greater ratio of hits to times at bat.

Example 3

Linda hits safely 5 out of 8 times at bat. Janet hits safely 7 out of 10 times at bat. Who has the greater ratio of hits to times at bat?

There are several ways to compare ratios with different first or second terms.
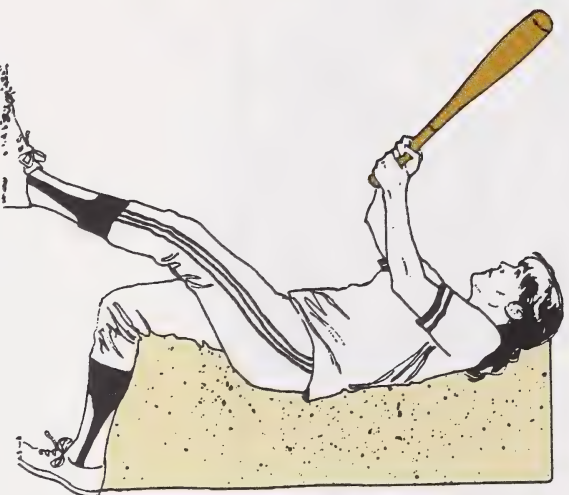
Method 1

First write the ratios of hits to times at bat.

$$\overbrace{\begin{array}{c} number\ of\ hits \\ number\ of\ times\ at\ bat \end{array}}$$
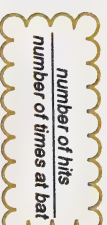
	Janet	Linda
	$\frac{7}{10}$	$\frac{5}{8}$

Then find equivalent ratios with the same second terms.

Janet

$$\frac{7}{10} \overset{\times 4}{=} \frac{28}{40} \underset{\times 4}{}$$

Linda

$$\frac{5}{8} \overset{\times 5}{=} \frac{25}{40} \underset{\times 5}{}$$

Because the second terms are the same, you can now compare the first terms.

$$28 > 25$$

So, $\frac{28}{40} > \frac{25}{40}$

and $\frac{7}{10} > \frac{5}{8}$.

Therefore, Janet has the greater ratio of hits to times at bat.

Method 2

First write the ratios of hits to times at bat.

$$\overbrace{\frac{number\ of\ hits}{number\ of\ times\ at\ bat}}$$

Janet	Linda
$\frac{7}{10}$	$\frac{5}{8}$

Then use the product of the second terms to write equivalent ratios.

Janet   Linda

$$\frac{7}{10} \overset{\times 8}{\underset{\times 8}{=}} \frac{56}{80} \qquad \frac{5}{8} \overset{\times 10}{\underset{\times 10}{=}} \frac{50}{80}$$

Then compare the first terms.

$$56 > 50$$

So, $\frac{56}{80} > \frac{50}{80}$

and $\frac{7}{10} > \frac{5}{8}$.

Therefore, Janet has the greater ratio of hits to times at bat.

Method 3

First write the ratios of hits to times at bat.

$$\overbrace{\frac{number\ of\ hits}{number\ of\ times\ at\ bat}}$$

Janet	Linda
$\frac{7}{10}$	$\frac{5}{8}$

Find cross products. This is a short-cut method to find the first terms in Method 2.

$$\overbrace{7 \times 8 = 56} \qquad \overbrace{5 \times 10 = 50}$$

$$\frac{7}{10} \times \frac{5}{8}$$

Then compare the cross products.

$$56 > 50$$

So, $\frac{7}{10} > \frac{5}{8}$.

So, Janet has the greater ratio of hits to times at bat.

Change the ratios into their decimal number forms by dividing.

A calculator can also be used to find the decimal number forms.

Janet

Janet

$$\frac{0.7}{10)\overline{7.0}}$$
$$\underline{7.0}$$

$$\frac{7}{10} = 0.7$$

Key Press	Display
7 ÷ 1 0 =	0.7

Linda

Linda

$$\begin{array}{r} 0.625 \\ 8)\overline{5.000} \\ \underline{4\ 8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \end{array}$$

$$\frac{5}{8} = 0.625$$

Key Press	Display
5 ÷ 8 =	0.625

Then compare the decimal numbers.

Then compare the decimal numbers.

0.7 = 0.700

0.7 > 0.625

0.7 > 0.625

So,

$$\text{So,}\quad \frac{7}{10} > \frac{5}{8}.$$

So,

$$\frac{7}{10} > \frac{5}{8}.$$

Therefore, Janet has the greater ratio of hits to times at bat.

Linda has the greater ratio of hits to times at bat.

Introductory Activities

1. Look at the photographs. Compare the ratios of the lengths to the widths. Are they equivalent?

10 cm

8 cm

5 cm

4 cm

2. Look at these cylinders. Compare the ratios of the heights to the diameters. Are they equivalent?

3 cm

2 cm

2 cm

3 cm
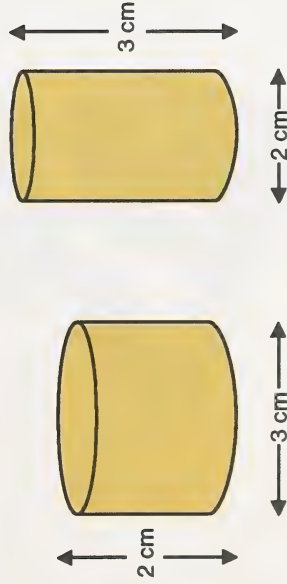
3. Last year a bowling team won 90 out of 162 games. So far this year they have won 10 out of 12 games. Is the team doing better or worse than it did last year?
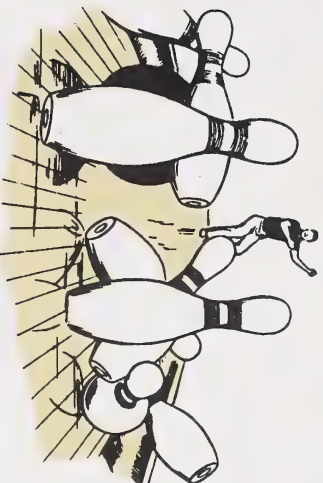
4. In four games a basketball player made 7 out of 10 shots, 12 out of 20 shots, 3 out of 5 shots, and 3 out of 4 shots.

a. In which game did the player have the greatest success?

b. In which game did the player have the least success?

c. In which games did the player have the same amount of success?

5. Matt and Jon play wheelchair basketball. Matt sank 6 out of 9 basketball shots. Jon sank 9 out of 15. Which of these two players is the more accurate shooter?
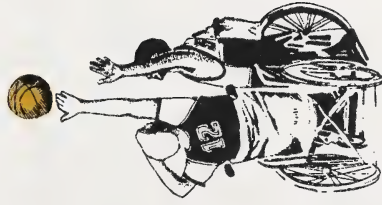
6. Kara scored 17 out of 25 on the first test and 14 out of 20 on the second test. On which test did she do better?

Computer Alternative

Space for Your Work

7. Do Lesson 7 of the disk *Ratio and Percent* from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA). Read the instructions in the folder with the disk before using the program.

Remember, if you need help, press the SHIFT key and the ? key.

See your learning facilitator to check your answers and to receive further instructions.

Working Together

In the Introductory Activities you compared ratios.

Scientists and mathematicians have been comparing ratios for a long time. They have made some interesting discoveries about insects.

- The ratio of the distance a grasshopper can jump to the grasshopper's body length is about 20 : 1.

- The ratio of the distance a flea can jump to the flea's body length is about 200 : 1.

- The ratio of the distance a cricket can jump to the cricket's body length is about 30 : 1.

Equivalent ratios are sometimes called **proportional ratios.**

Scientists have discovered other proportional ratios.

- The ratio of the mass an ant can lift to its own body mass is 50 : 1.

- The ratio of mass a bee can pull to the bee's own body mass is 300 : 1.

Practice Activities

Space for Your Work

1. Consider the following rectangles.

   A B C D E

   a. Which of these appeals most to your eye?

   b. Measure the length and width of each rectangle. Use your calculator to find the ratio of length to width for each. Give each ratio in its decimal number form rounded to the nearest tenth.

   c. The ancient Greeks felt that the rectangles that were most appealing to the eye were those with ratios of length to width of 1.618. They called this the **golden ratio**. They used rectangles with this ratio in many of their buildings.

   Are any of the ratios in Part b. of this question close to the golden ratio? If so, which ones?

2. a. Recruit three friends and measure their heights from the floor to their shoulders. Then measure the heights of their heads from their chins to the tops of their heads. Use these measurements to find the ratio of the height of the body to the height of the head.

	Person 1	Person 2	Person 3
Height of Body			
Height of Head			
Ratio of Height of Body to Height of Head			
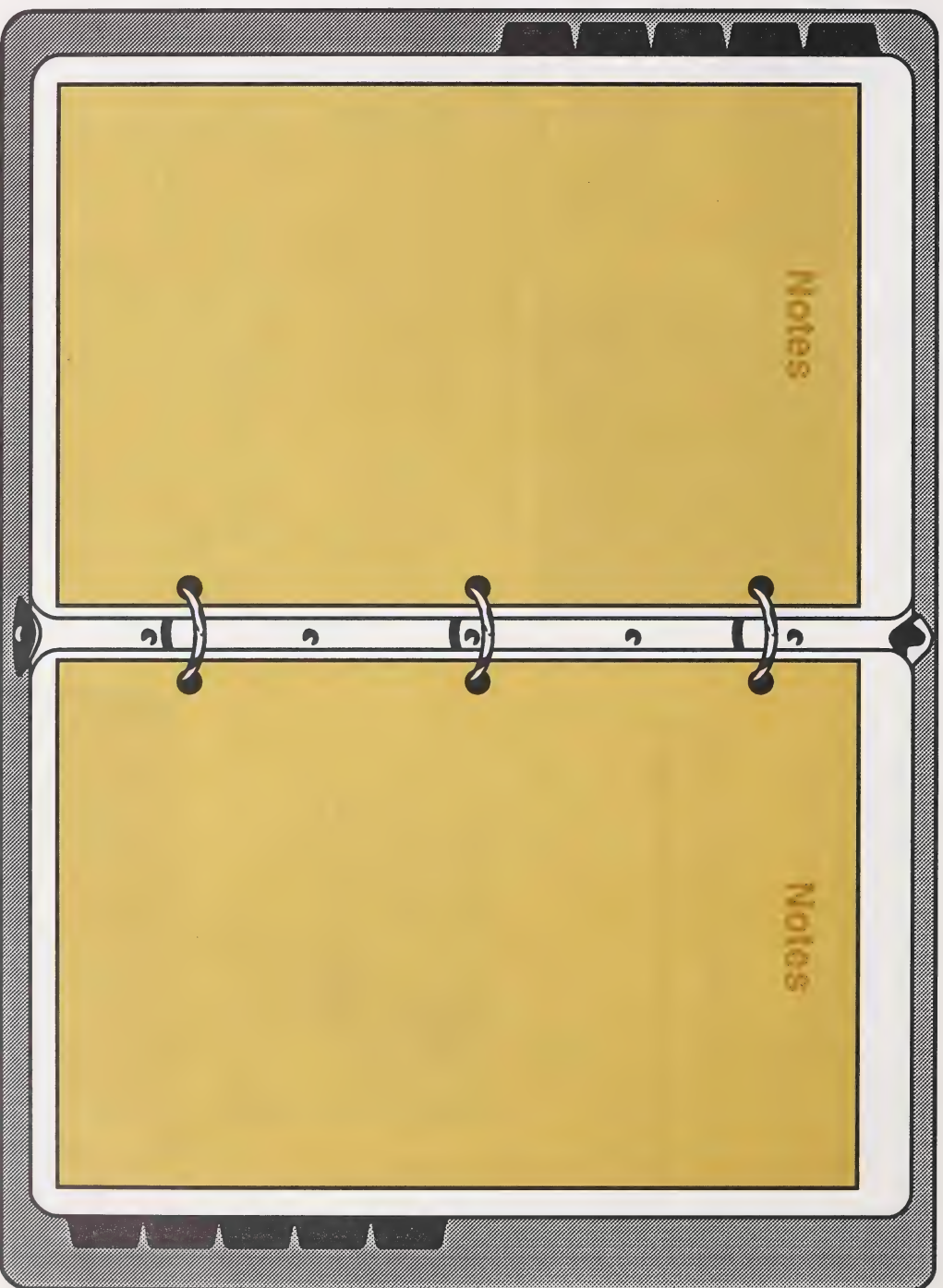
b. What do you notice about the ratios in Part a.?

c. Examine the cartoon character below. What do you notice?

See your learning facilitator to check your answers and to receive further instructions.

Notes

Notes

What Lies Ahead

In this section you will learn these skills.

- writing a proportion
- finding the missing term of a proportion

In this section you will learn this term.

- proportion

Working Together

In the previous section you learned about equivalent or proportional ratios.

A **proportion** is an equation that shows the equality of two ratios.

Proportions are very helpful. They allow you to find a missing term of a ratio if you are given one term of the ratio and a proportional ratio.

Example 1

Cal measured the shadow of a metre stick and the shadow of a flagpole at the same time of day. The shadow of the metre stick was 3 m long. The shadow of the flagpole was 15 m long. How high is the flagpole?

Method 1

Write the proportion. Use a box for the missing term.

$$\frac{\text{height of object in m}}{\text{length of its shadow in m}}$$

$$\begin{array}{ccc} & \text{metre stick} & \text{flag pole} \\ & \dfrac{1}{3} & = & \dfrac{\boxed{}}{15} \end{array}$$

Find the missing term in the proportion. This can be done using multiplication.

$$\overset{\times 5}{\underset{\times 5}{\frac{1}{3}}} = \frac{\boxed{5}}{15}$$

The flagpole is 5 m high.
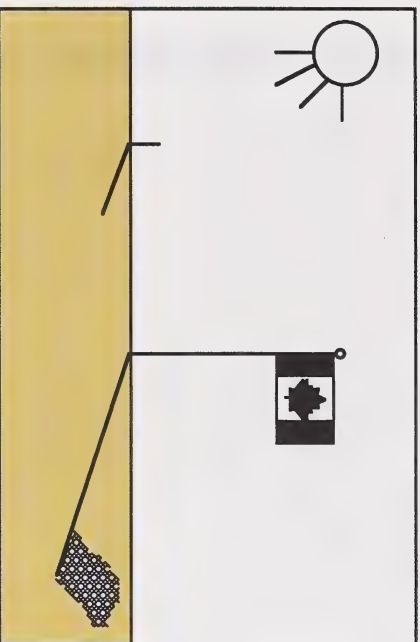
Solution

There are three ways to solve this problem. All three ways involve writing a proportion.

Remember that when you write a proportion, it is very important that you put the terms in the correct order.

Method 2

Write the proportion. Use a variable for the missing term.

$$\frac{\text{height of object in m}}{\text{length of its shadow in m}}$$

metre stick   flagpole

$$\frac{1}{3} = \frac{h}{15}$$

Find the cross product.

$1 \times 15 = 15$

$$\frac{1}{3} \diagdown\!\!\!\diagup \frac{h}{15}$$

$3 \times h = 3h$

Since this is a proportion, the cross products are equivalent.

$$3h = 15$$

Multiply both sides of the equation by $\frac{1}{3}$.

$$\frac{1}{\cancel{3}} \times \cancel{3}h = \frac{1}{\cancel{3}} \times \cancel{15}^{5}$$

$$h = 5$$

So, the height of the flagpole is 5 m.

Method 3

Write the proportion. Use a variable for the missing term.

$$\frac{\text{height of object in m}}{\text{length of its shadow in m}}$$

metre stick   flagpole

$$\frac{1}{3} = \frac{h}{15}$$

Multiply both sides of the equation by 15.

$$\cancel{15}^{5} \times \frac{1}{\cancel{3}} = \cancel{15}^{1} \times \frac{h}{\cancel{15}_{1}}$$

$$5 = h$$

So, the height of the flagpole is 5 m.

Example 2

As Minal walks with her little sister, Kana, she takes 4 steps for every 5 steps that Kana takes. How many steps will Kana have to take if Minal takes 50 steps?

Solution

There are three ways to solve the problem. All three ways involve writing a proportion.

Remember that when you write a proportion you must put the terms in the correct order.

Method 1

Write the proportion. Use a box for the missing term.

$$\frac{Minal's\ steps}{Kana's\ steps} \qquad \frac{4}{5} = \frac{50}{\blacksquare}$$

Find the missing term in the proportion.

4 times what = 50?
4 times 12 = 50

$$\frac{4}{5} \overset{\times\ 12}{\underset{\times\ 12}{=}} \frac{50}{\boxed{60}}$$

So, Kana takes about 60 steps for Minal's 50 steps.

Method 2

Write the proportion. Use a variable for the missing term.

Minal's steps / Kana's steps

$$\frac{4}{5} = \frac{50}{k}$$

Find the cross product

$4 \times k = 4k$

$5 \times 50 = 250$

$$\frac{4}{5} \diagup\!\!\!\!\diagdown \frac{50}{k}$$

Since this is a proportion, the cross products are equivalent.

$$4k = 250$$

Multiply both sides of the equation by $\frac{1}{4}$.

$$\frac{1}{\cancel{4}} \times \cancel{4}\, k = \frac{1}{\cancel{4}} \times \overset{62.5}{\cancel{250}}$$

$$k = 62.5$$

So, Kana takes 62.5 steps when Minal takes 50 steps.

Method 3

Write the proportion. Use a variable for the missing term.

Minal's steps / Kana's steps

$$\frac{4}{5} = \frac{50}{k}$$

Multiply both sides of the equation by $5k$.

$$\cancel{5}\,k \times \frac{4}{\cancel{5}} = \cancel{5}\,k \times \frac{50}{\cancel{k}}$$

$$4k = 250$$

Multiply both sides of the equation by $\frac{1}{4}$.

$$\frac{1}{\cancel{4}} \times \cancel{4}\, k = \frac{1}{4} \times 250$$

$$k = 62.5$$

Note

Methods 2 and 3 are preferable because they give more accurate answers.

Practice Activities

Space for Your Work

1. The ratio of the weight of an object on the moon to the weight of an object on Earth is 1 to 6. What is the moon weight of these objects that have been weighed on Earth?

    a.   a 42-kg boy

    b.   a 3-kg cat

    c.   a 0.05-kg mouse

2. The moon pulls objects towards it with a different amount of force than the earth does. It is easier to jump away from the surface of the moon than it is to jump away from the surface of the earth. The ratio of a person's jump on Earth to a person's jump on the moon is 1 to 6.

    a.   If a high jumper can jump 2.25 m on Earth, how high would the jump be on the moon?

    b.   If a pole jumper can jump 5.5 m on Earth, how high would the jump be on the moon?

3. Your body contains 206 bones. The ratio of the number of bones in your head to the total number of bones in your body is about 1 to 7. About how many bones are there in your head?

4. The ratio of the length of your small intestine to the total length of both your large and small intestine is 4 : 5. If your large and small intestines are about 7.6 m long in total, how long is your small intestine?
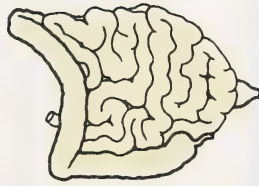
See your learning facilitator to check your answers and to receive further instructions.

Working Together

You can use proportions to find a missing term in a three-term ratio.

Example 1

If stereo speakers are to have good acoustics (sound), the ratio of depth to width to height should be 1 : 2 : 3. Richard made his speakers 60 cm wide. What depth and height are the speakers?

Solution

There are two ways to solve this problem.
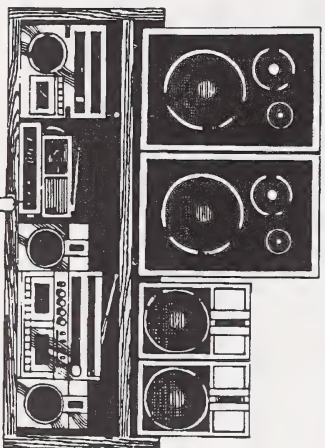
Method 1

Write the three-term ratios as a proportion. Use boxes for the unknown terms in the second ratio.

depth : width : height

$$1 : 2 : 3$$
$$= \blacksquare : 60 : \bullet$$

By comparing, you can see that 2 was multiplied by 30 to get 60. Therefore, the other known terms may also be multiplied by 30 to solve for the unknown terms.

```
        1  :  2  :  3
        |     |     |
      ×30   ×30   ×30
        |     |     |
      ↓       ↓     ↓
   ▓▓   :  60  :  ●
```

By this method you find that the depth (▓▓) is 30 cm and the height (●) is 90 cm.

Method 2

Write the three-term ratio as a proportion. Use variables for the unknown terms in the second ratio.

$$\text{depth : width : height}$$

$$1 : 2 : 3$$

$$= d : 60 : h$$

To find the depth, write a proportion using the first and second terms of each ratio.

$$\frac{1}{d} = \frac{2}{60}$$

Find the cross product and solve.

$$2d = 60$$

$$\frac{1}{2} \times 2d = \frac{1}{2} \times 60$$

$$d = 30$$

The depth (d) is 30 cm.

To find the height, write a proportion using the second and third terms of each ratio.

$$\frac{2}{60} = \frac{3}{h}$$

Find the cross product and solve.

$$2h = 180$$

$$\frac{1}{2} \times 2h = \frac{1}{2} \times 180$$

$$h = 90$$

The height (h) is 90 cm.

Example 2

The number of parts of red paint to yellow paint to black paint in a particular shade of brown paint is 8 : 8 : 1.

If you want to make 1000 mL of brown paint, how much red paint would you use?
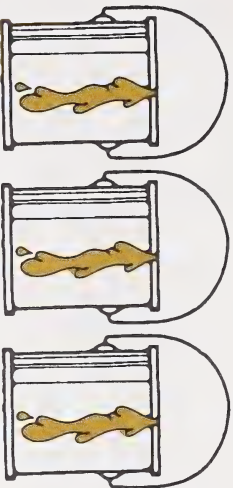
Solution

There are two ways to solve this problem.

Method 1

Find the ratio of the number of parts of red paint to brown paint.

$$8 + 8 + 1 = 17$$

So, the ratio of the number of parts of red paint to brown paint is 8 : 17.

Now write a proportion. Use a box for the missing term.

red paint (mL)
brown paint (mL)

$$\frac{8}{17} = \frac{\blacksquare}{1000}$$

Find the missing term in the proportion.

17 times what = 1000?
17 times 60 = 1000

$$\frac{8}{17} = \frac{\boxed{480}}{1000}$$

×60

×60

So, about 480 mL of red paint is needed to make 1000 mL of brown paint.

Method 2

Find the ratio of the number of parts of red paint to brown paint.

$$8 + 8 + 1 = 17$$

So, the ratio of the number of parts of red paint to brown paint is 8 : 17.

Write the proportion using a variable for the missing term.

$$\frac{\text{red paint (mL)}}{\text{brown paint (mL)}} \quad \frac{8}{17} = \frac{r}{1000}$$

Find the cross product and solve.

$$17r = 8000$$

$$\frac{1}{17} \times 17r = \frac{1}{17} \times 8000$$

$$r \doteq 470.6$$

So, the amount of red paint needed is about 470.6 mL.

Note

Method 2 is preferable because it gives a more accurate answer.

Concluding Activities

1. Three partners invest in a company in the ratio of 5 : 4 : 1. If they divide their profits of $150 000 in the same ratio as they invested, how much does each partner receive?

2. The ratio of the number of red marbles to blue marbles to white marbles is 5 : 12 : 8. If there are 100 marbles in the bag, how many marbles of each colour are there?

See your learning facilitator to check your answers and to receive further instructions.

SUMMARY

What Lies Ahead

In this section you will review the skills you have learned in Sections 1 to 5.

- interpreting a ratio

- writing a ratio using the colon form, the fraction form, and the decimal number form

- writing equivalent ratios

- writing ratios in simplest form

- comparing and ordering ratios

- writing a proportion

- finding the missing term for a ratio in a proportion

Working Together

Turn to Section 1 in this Module Booklet and correct any errors you may have made in the pretest. You may be pleasantly surprised to discover how much you have learned about ratios!

Rates are commonly used in the everyday world. You have probably heard some of the following expressions of rate.

- the rate of speed

- the gas-consumption rate

- the pulse rate

- the rate of inflation

- the rate of population growth

This part of the module will build on what you already know about rates.

WESTFILE INC.

GETTING SET

What Lies Ahead

This section will test your knowledge of these skills.

* writing rates using the colon form and the fraction form

* simplifying rates

* solving rate problems using proportions

Working Together

Sections 7 to 10 deal with rates.

The pretest in this section will help you and your learning facilitator determine your strengths and weaknesses.

Pretest

1. a. Raja can type 99 words in 4 minutes. Express this as a rate. Use the colon form.

   b. Shawn sold 39 tires in 7 hours. Express this as a rate. Use the fraction form.
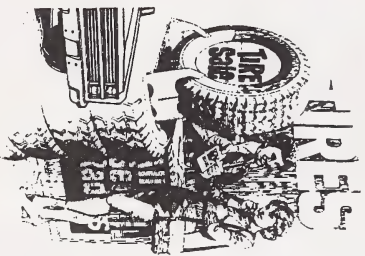
Space for Your Work

2. It rained 42 mm in 10 h. What is the rate of rainfall in simplest form?

3. Francena bought 9 oranges. If the selling price of the oranges is 12 for $3.99, what did Francena pay?

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

RATES

Working Together

In Part One you did not write the units as a part of the ratio because the quantities being compared were measured in the same unit.

In this section you will learn about special ratios called **rates** that compare quantities with different units.

What Lies Ahead

In this section you will learn these skills.

- reading and writing rates

- simplifying rates

Speed rates express the ratio of distance to time. Because the units are different, they must be included in the ratio.

Example

The bus from Edmonton to Calgary travels a distance of 300 km in 3 h.
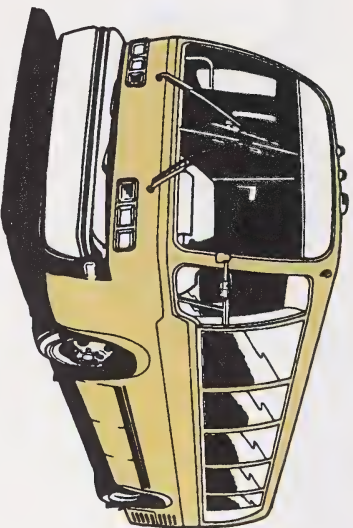
The rate of speed can be expressed in different ways.

- The rate of speed is 300 km : 3 h.

- The rate of speed is 300 km/3 h.

- The rate of speed is $\frac{300 \text{ km}}{3 \text{ h}}$.

These expressions are all read as "300 km per 3 h."

Price rates express the ratio of price to mass. Because the units are different, they must be included in the ratio.

Example

The cost of 0.865 kg of chicken is $5.18.

The price rate can also be expressed several ways.

- The price rate is $5.18 for 0.865 kg.

- The price rate is $5.18 : 0.865 kg.

- The price rate is $5.18/0.865 kg.

- The price rate is $\frac{\$5.18}{0.865 \text{ kg}}$.

These expressions are all read as "$5.18 per 0.865 kg."

Introductory Activities

Space for Your Work

1. Write rates to describe the following situations. Use the colon form.

   a. A child buys 2 apples for $0.80.

   b. A typist types 700 words in 8 minutes.

   c. A metal expands 12.7 cm as the temperature rises 18° C.

   d. A spring stretches 10.5 cm when a 150-g mass is hung on it.

   e. A person breathes 1500 times in 24 hours.

2. Write ratios to describe the statements in Question 1. Use the fraction form.

> See your learning facilitator to check your answers and to receive further instructions.

Working Together

Usually rates are simplified. One way to simplify a rate is to write the rate with a second term of 1.

Example 1

It rained 81.6 mm in 5 h. What was the hourly rate of rainfall?

Solution

Write the rate of rainfall in 5 hours.

$$81.6 \text{ mm/5 h}$$

Then find the hourly rate by dividing each term by 5.

$$\frac{81.6 \text{ mm} \div 5}{5 \text{ h} \div 5} = \frac{16.32 \text{ m}}{1 \text{ h}}$$

The rate of rainfall is 16.32 mm/h.

Example 2

Joseph earns $138 for 8 hours of work. What is Joseph's hourly rate of pay?

Solution

Write Joseph's rate of pay for 8 hours.

$$\$138/8 \text{ h}$$

Then find Joseph's hourly rate of pay by dividing each term by 8.

$$\frac{\$138 \div 8}{8 \text{ h} \div 8} = \frac{\$17.25}{1 \text{ h}}$$

Joseph's rate of pay is $17.25/h.

Writing rates with a second term of 1 makes it easier to compare the rates.

Example

Garry earned $56 in 8 hours. Diane earned $48 in 6 hours. Who earned more money per hour?

Solution

Write Garry's rate of pay for 8 h.

$56/8 h

Calculate Garry's hourly rate of pay.

$$\frac{\$56 \div 8}{8\,h \div 8} = \frac{\$7}{1\,h}$$

Garry earned $7/h.

Write Diane's rate of pay for 6 h.

$48/6 h

Calculate Diane's hourly rate of pay.

$$\frac{\$48 \div 6}{6\,h \div 6} = \frac{\$8}{1\,h}$$

Diane earned $8/h.

$8/h > $7/h.

So, Diane earned more money per hour.

Practice Activities

1. Which is the better buy, 5 kg of bananas for $7.35 or 3 kg of bananas for $4.45?

2. Dino travelled 830 km in 9 h. Frank travelled 640 km in 7 h. Who travelled at a faster rate of speed?

3. An athlete runs 100 m in 9.9 seconds. Another athlete runs 200 m in 19.8 seconds. Which athlete runs faster?

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Example

Fuel consumption is measured in litres per hundred kilometres.

Example

George's car used 68 L in a 500-km trip. What is his rate of fuel consumption?
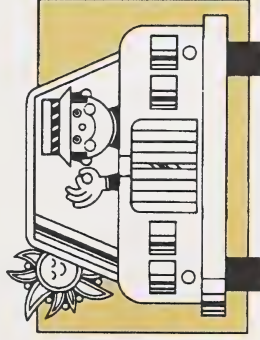
Solution

The rate is 68 L/500 km. Calculate the litres per 100 km.

$$\frac{68\,L \div 5}{500\,km \div 5} = \frac{13.6\,L}{100\,km}$$

George's gas consumption is 13.6 L/100 km.

Some other costs are measured in dollars per hundred millilitres or dollars per hundred grams.

Example

Shampoo costs $3.79 for 450 mL. What is the cost per 100 mL?

SUDS
SHAMPOO
450 ml
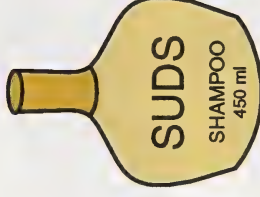
Solution

The cost is $3.79/450 mL.

Calculate the cost per 100 mL.

$$\frac{3.79}{450} = \frac{n}{100}$$

$$450n = 379$$

$$n = 0.84$$

The shampoo costs $0.84/100 mL.

Concluding Activities

1. Manuel's car travels 800 km on 72 L of gas. Ria's car travels 350 km on 33 L of gas. Who gets the better rate of gasoline consumption? Express the rates as L/100 km.
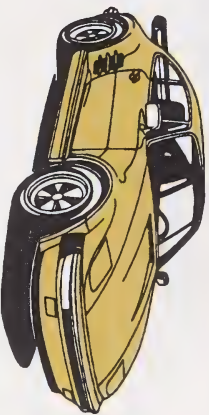
2. A 750-mL bottle of pop sells for $1.29. A 2-L bottle of pop sells for $3.05. Which is the better buy? Express the rates as $/100 mL.
   (Hint: 1 L = 1000 mL.)

See your learning facilitator to check your answers and to receive further instructions.

Space for Your Work

USING PROPORTIONS TO SOLVE RATE PROBLEMS

What Lies Ahead

In this section you will learn these skills.

- writing a proportion involving rates

- solving rate problems using proportions

Working Together

In Section 5 you used proportions to solve problems involving ratios. In this section you will use proportions to solve problems with rate.

Example

Sandra is watching a storm. She sees lightning and 8 s later she hears thunder. How far away is the lightning? (Hint: Sound travels at a rate of 344 m/s.)

Solution

There are two ways to solve this problem.

Method 1

Write the rate of speed at which sound travels.

344 m/s

Use this rate to write a proportion. Use a box for the missing term.

$\dfrac{m}{s}$

$$\frac{344}{1} = \frac{\blacksquare}{8}$$

Multiply both terms by 8 to find the missing term.

$$\frac{344}{1} \overset{\times 8}{=} \frac{2752}{8}\ _{\times 8}$$

The lightning is 2752 m away.

Method 2

Write the rate of speed at which sound travels.

344 m/s

Use this rate to write a proportion. Use a variable for the missing term.

$\dfrac{m}{s}$

$$\frac{344}{1} = \frac{d}{8}$$

Use cross products to solve for the missing term.

$$d = 2752$$

The lightning is 2752 m away.
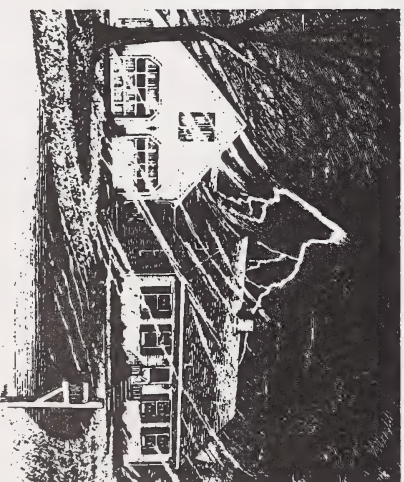
Practice Activities

1. Arthur, in training for a 10-km race, jogs 6 km daily. It takes him an average of 30 minutes to jog this distance.

    a. Write the rate that represents Arthur's training rate.

    b. At this rate, how long will it take him to finish the 10-km race? Write the proportion and solve.

    c. How far can he jog in 45 minutes?

2. Complete the following table.

Kilograms of Meat	2	4		9.5
Cost in Dollars	8.60		43.00	

3. If 8 pairs of socks sell for $10.99, what do 10 pairs cost?

4. At a rate of 2.5 m/s, how far can Arlene swim in 50 s?

See your learning facilitator to check your answers and to receive further instructions.

Space for Your Work

Extra Practice

Space for Your Work

1. The human heart beats an average of 72 times per minute. At this rate, how many heartbeats are there in the following time periods?

    a. 5 min

    b. 30 min

    c. 45 min

2. The table below lists the cruising speeds of three planes.

Type of Plane	Cruising Speed (km/h)
DC-9	826
747	893
L-1011	882

    For each plane, determine how far it can travel in 3 h.

3. René, who works a 40-h week, is paid $450 per week. At the same hourly wage, how much will she receive if she works a 36-h week?

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

1. Geoff can run at 10 m/s. The world's fastest human can run at 7.5 m/s. If Geoff had a head start of 30 m, would he win a 100-m race?

2. A swimmer's best record for the 100-m freestyle is 51.2 seconds. Her best record for the 200-m freestyle is 1 minute 52.8 seconds. If she could swim the 200-m race at the same rate as the 100-m race, how much time would she cut from her record?

✓ See your learning facilitator to check your answers and to receive further instructions.

SUMMARY

What Lies Ahead

In this section you will review the skills you have learned in Sections 7 to 10.

Working Together

At this time it is a good idea to review Part Two. Turn to Section 7 and correct any errors you may have made in the pretest. You may be pleasantly surprised to discover how much you have learned about rates!

Percents are special ratios. You probably are familiar with percents, as they are used frequently in the everyday world.

For example, the amount of milk fat in dairy products is expressed in percents.

• Partly-skimmed milk is 2% milk fat.

• Whipping cream is 35% milk fat.

WESTFILE INC.

GETTING SET

What Lies Ahead

In this section you will test your knowledge of these skills.

- interpreting percents less than 100%

- interpreting percents greater than 100%

- expressing a percent as a decimal number and a fraction

- expressing a decimal number and a fraction as a percent

- using proportions to solve problems of this form: What percent is a of b?

- finding a percent of a number

- finding percent increases and decreases

Working Together

Part Three deals with percents. The following pretest will help you and your learning facilitator determine your strengths and weaknesses.

Pretest

Space for Your Work

1. In a package of 100 gummed stars there are 28 blue, 13 gold, 23 green, and 15 silver stars. The rest are red.

   a. Express the number of stars of each colour as a percent of the total.

   b. Express the number of green stars as a percent of the gold stars.

   c. Express the number of blue stars as a percent of the silver stars.

3. There are 9 eggs in a carton that holds 12 eggs when full.

   a. What percent of the egg carton is full?

   b. What percent of the egg carton is empty?

4. Rewrite each of these sentences using a percent.

   a. The team won 0.85 of its games.

   b. During a sale the price was reduced by $\frac{1}{2}$.

   c. Maria scored 18 out of 25 on a test.

   d. The cost of the book is 2.5 times its price ten years ago.

   e. Dave needed $1\frac{1}{2}$ times the number of bricks that he had to build a wall.

5. Express each decimal number as a percent.

a. 0.35

b. 0.07

c. 4.2

d. 1.62

6. Express each fraction as a percent.

a. $\frac{23}{100}$

b. $\frac{3}{10}$

c. $\frac{4}{5}$

d. $2\frac{3}{20}$

e. $1\frac{7}{25}$

Space for Your Work

7. Express each percent as a decimal number and as a fraction in lowest terms.

    a. 45%

    b. 9%

    c. 200%

    d. 160%

8. Estimate the answer for each of the following.

    a. 40% of 102

    b. 22% of 10

    c. 66% of 300

    d. 125% of 400

9. Calculate the answer for each of the following.

   a. 48% of 2000

   b. 22% of 10

   c. 66% of 300

   d. 130% of 500

10. A store is holding a 25% off sale. What is the sale price of a sweater that regularly sells for $45.98?

11. In Alberta a goods and services tax of 7% is added to the price of clothing. What is the final price of a pair of jeans that sells for $39.99?

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will learn these skills.

* interpreting percents less than 100%

* expressing a percent as a decimal number and as a fraction

* expressing a fraction or a decimal number as a percent

Working Together

Percents are special ratios. **Percent** means *out of 100* or *for every 100*. The second term in a percent is always 100, but it is not written. Instead, the % symbol is used.
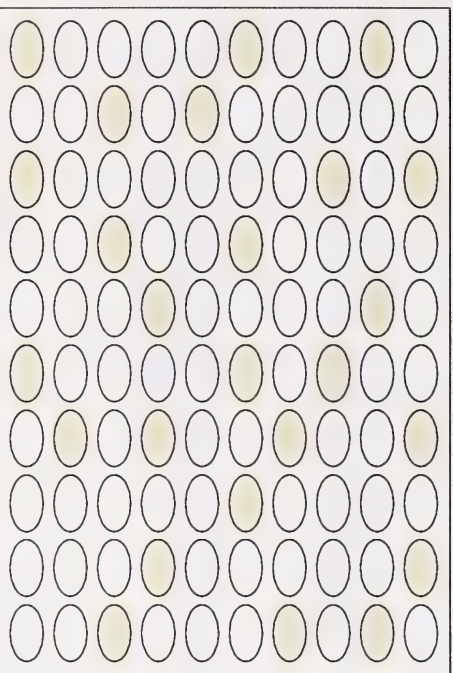
90% = 90 : 100

This is read as "90 percent."

Expressing Percents as Fractions and Decimal Numbers

A percent can be expressed as a fraction or a decimal number.

Example 1

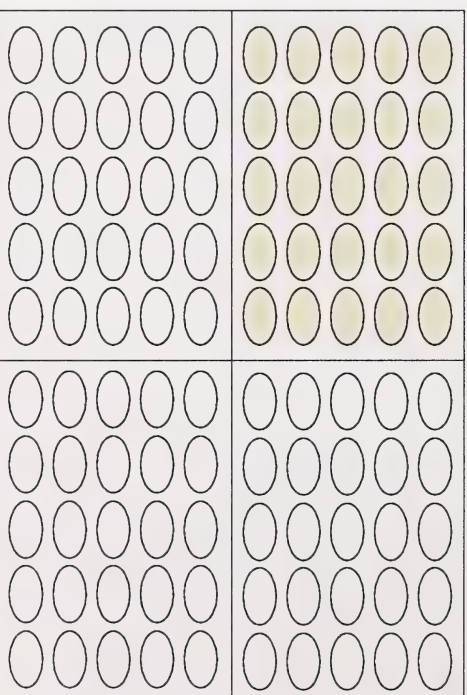25% of the eggs shown are brown eggs.

25% means that 25 eggs out of 100 eggs are brown.

25% can be expressed as a fraction.

$$25\% = \frac{25}{100}$$

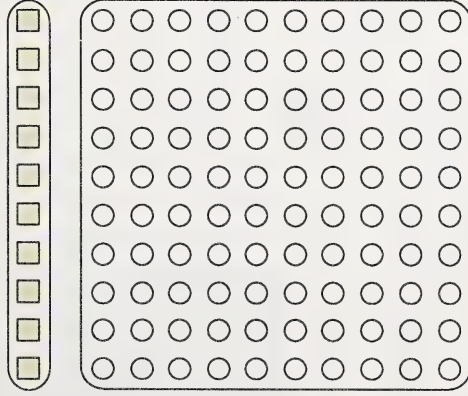By rearranging the eggs, you can see that $\frac{1}{4}$ of the eggs are brown.

$$25\% = \frac{1}{4}$$

You already know that $\frac{1}{4} = 0.25$.

So, 25% = 0.25.

Example 2

The number of squares is 10% of the number of circles.
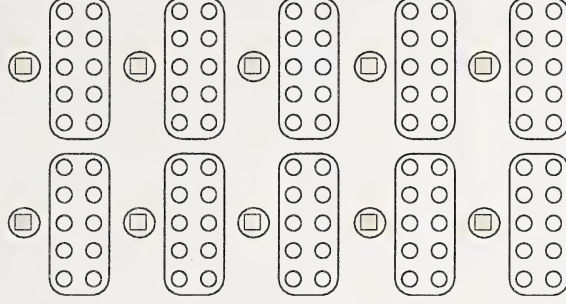

10% means that there are 10 squares for every 100 circles.

10% can be expressed as a fraction.

$$10\% = \frac{10}{100}$$

By rearranging the squares and circles, you can see that the number of squares is $\frac{1}{10}$ of the number of circles.

So, $10\% = \frac{1}{10}$.

You already know that $\frac{1}{10} = 0.1$.

So, $10\% = 0.1$.

Example 3

You can purchase bread that is 60% whole wheat. This means that 60 parts out of 100 parts of flour are whole wheat.
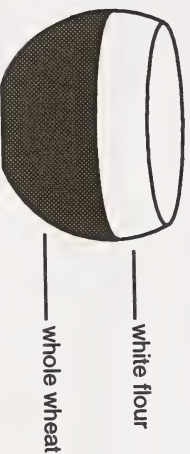

$$60\% = \frac{60}{100}$$
$$= 0.60$$
$$= 0.6$$

$$\frac{60}{100} = \frac{6}{10} = \frac{3}{5}$$

This means 0.6 of the flour is whole wheat.

You have already learned that $0.6 = \frac{6}{10} = \frac{3}{5}$.

—— white flour

—— whole wheat
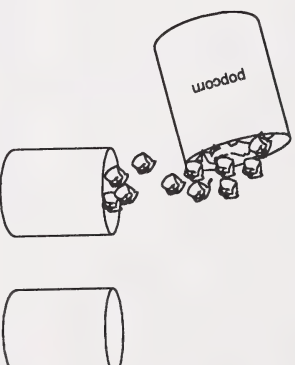
This means that $\frac{3}{5}$ of the flour is whole wheat.

Example 4

This container of popcorn has 100% of the popcorn.
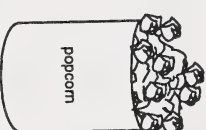
The popcorn can be divided into smaller containers.

Now, each of the smaller containers has 50% of the popcorn.

$$50\% = \frac{50}{100}$$
$$= \frac{1}{2}$$

This means that each container has $\frac{1}{2}$ of the popcorn. You already know that $\frac{1}{2} = 0.5$.

Example 5
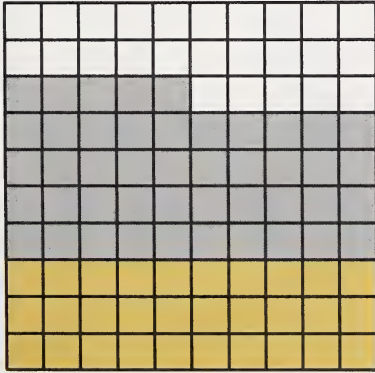
30% of the squares are brown.

45% of the squares are grey.

25% of the squares are white.

Notice that the total of the whole is 100%.

30% + 45% + 25% = 100%

One of the advantages of expressing ratios as percents is that they are easy to compare.

	Percent	Fraction	Decimal
Brown	30%	$\frac{3}{10}$	0.3
Grey	45%	$\frac{9}{20}$	0.45
White	25%	$\frac{1}{4}$	0.25
Total	100%	1	1

Space for Your Work

Practice Activities

1. A loaf of bread is 76% whole wheat. Express the percent of whole wheat flour in this loaf of bread as each of the following.

   a. a fraction in lowest terms
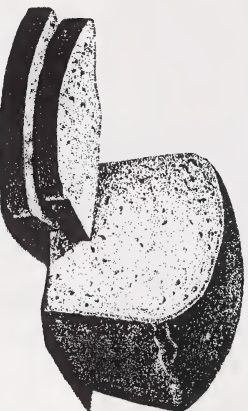
   b. a decimal number

2. The label on a sweater shows that the sweater is made of wool and acrylic fibre. Express each part as the following.

   a. a fraction in lowest terms

   b. a decimal number

45% wool
55% acrylic fibre
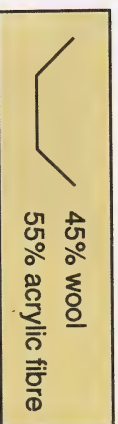
3. Coffee cream is 18% milk fat. Express the percent of milk fat as each of the following.

   a. a fraction in lowest terms

   b. a decimal number

4. The number of red balloons in a package is 40% of the number of yellow balloons. Express the percent as each of the following.

   a. a fraction in lowest terms
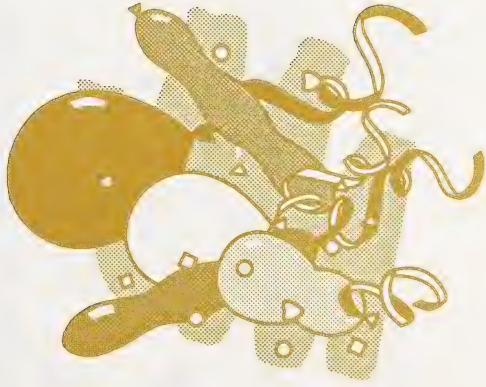
   b. a decimal number

Space for Your Work

5. A metal alloy is made of zinc, tin, and lead. The amount of tin is 60% of the amount of lead. Express this percent as each of the following.

a. a fraction in lowest terms

b. a decimal number

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Expressing a Fraction as a Percent and as a Decimal Number

Example 1

An alloy of copper, zinc, and lead is used in making clock parts. $\frac{16}{25}$ of the alloy is copper. Express this fraction as a percent and as a decimal number.

Solution

To write the fraction as a percent, you must first find an equivalent fraction that has 100 as its denominator.

$$\frac{16}{25} = \frac{\blacksquare}{100}$$

$$\frac{16}{25} \overset{\times 4}{=} \frac{\boxed{64}}{100}$$

Then write the equivalent fraction as a percent.

$$\frac{64}{100} = 64\%.$$

So, 64% of the alloy is copper.

To write the fraction as a decimal number, first write the equivalent fraction that has 10, 100, 1000, … as its denominator.

$$\frac{16}{25} = \frac{64}{100}$$

Now it should be easy to change to a decimal number.

$$\frac{16}{25} = \frac{64}{100} = 0.64.$$

This can be read as either "64 hundredths" or "0 point 64."

So, 0.64 of the alloy is copper.

Example 2

Mrs. Ricci's expenses are $\frac{4}{5}$ of Mrs. Morizama's expenses. Express the fraction as a percent and as a decimal number.

Solution

To find the percent, first find the equivalent fraction with 100 as the denominator.

$$\frac{4}{5} = \frac{\blacksquare}{100}$$

$$\overset{\times 20}{\overset{\frown}{\frac{4}{5}}} = \frac{\boxed{80}}{100} \underset{\times 20}{\underset{\smile}{}}$$

Write the equivalent fraction as a percent.

$$\frac{80}{100} = 80\%$$

Mrs. Ricci's expenses are 80% of Mrs. Morizama's expenses.

To write the original fraction as a decimal number, write the equivalent fraction that has 10, 100, 1000, ... as its denominator.
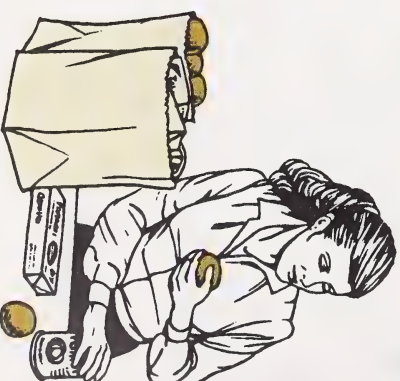
$$\frac{4}{5} = \frac{8}{10}$$

Now it should be easy to change to a decimal number.

$$\frac{8}{10} = 0.8$$

This is read as "8 tenths" or "0 point 8."

Mrs. Ricci's expenses are 0.8 of Mrs. Morizama's expenses.

Expressing a Decimal Number as a Percent and as a Fraction

Example 1

The 1982 Canadian gold coin is an alloy of gold and silver. 0.92 of the coin is gold. Express the decimal number as a percent and as a fraction.

Solution

To change the decimal number to a percent, first write the decimal number as a fraction with 100 as its denominator.

$$0.92 = \frac{92}{100}$$

Then write the fraction as a percent.

$$\frac{92}{100} = 92\%$$

92% of the coin is gold.

To change the decimal number to a fraction, first write it as a fraction that has 10, 100, 1000, ... as its denominator.

$$0.92 = \frac{92}{100}$$

Then write the fraction in lowest terms.

$$\frac{92}{100} = \frac{23}{25}$$

This is read as "23 twenty-fifths."

$\frac{23}{25}$ of the coin is gold.

Example 2

The amount of lemonade in a punch is 0.3 of the amount of ginger ale. Express the decimal number as a percent and as a fraction.

Solution

To change the decimal number to a percent, write the decimal number as a fraction.

$$0.3 = \frac{3}{10}$$

Then find the equivalent fraction that has 100 as its denominator.

$$\frac{3}{10} = \frac{30}{100}$$

Finally write the fraction as a percent.

$$\frac{30}{100} = 30\%$$

The amount of lemonade in the punch is 30% of the amount of ginger ale.

To change the decimal number to a fraction, write it as a fraction that has 10, 100, 1000, ... as its denominator.

$$0.3 = \frac{3}{10} \leftarrow \text{This is read as "3 tenths."}$$

The amount of lemonade in the punch is $\frac{3}{10}$ of the amount of ginger ale.
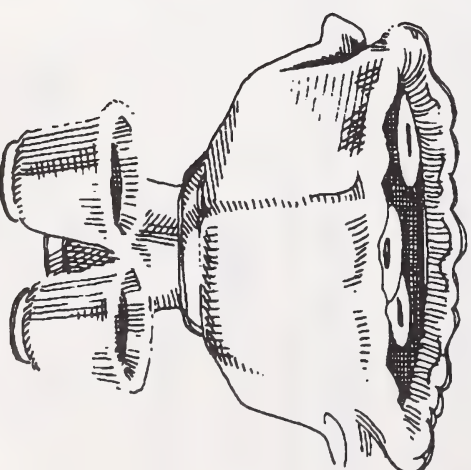
Practice Activities

1. About $\frac{3}{4}$ of the students in the computer class were present. Express the fraction as each of the following.

   a. a percent

   b. a decimal number

2. The baseball player hit safely 0.25 of the times he was up to bat. Express the decimal number as each of the following.

   a. a percent

   b. a fraction

3. The amount of fence Bruce painted is $\frac{1}{5}$ of the amount Joan painted. Express this fraction as each of the following.

a. a percent

b. a decimal number

4. Jake's age is 0.3 of his father's age. Express the decimal number as each of the following.

a. a percent

b. a fraction
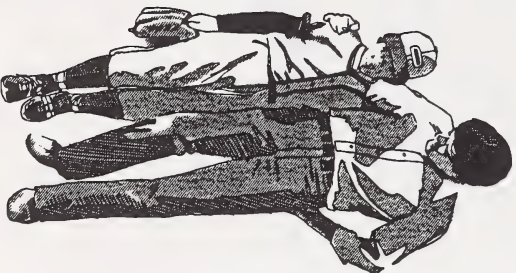
Space for Your Work

5. Complete this table. Make sure that you give the fractions in lowest terms.

Space for Your Work

	Fraction	Decimal Number	Percent
a.		0.01	
b.			2%
c.	$\frac{1}{10}$		
d.		0.35	
e.			20%
f.		0.4	
g.	$\frac{1}{2}$		
h.		0.6	

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

1. Write >, <, or = in each ⬤ . Use mental computation only.

   a. 100% ⬤ 1

   b. 25% ⬤ 2.5

   c. 50% ⬤ $\frac{1}{5}$

   d. 60% ⬤ 0.06

   e. 75% ⬤ $\frac{2}{3}$

   f. 10% ⬤ 10

   g. 4% ⬤ 0.4

   h. 1% ⬤ 1

Space for Your Work

2. You can use patterns to find the percent that is equivalent to a fraction.

Example

Fraction	Percent
$\frac{1}{2}$	50%
$\frac{2}{2}$	100%

$+\frac{1}{2}$ $+50\%$

Complete the following charts mentally by using patterns.

a.

Fraction	Percent
$\frac{1}{4}$	25%
$\frac{2}{4}$	
$\frac{3}{4}$	

Space for Your Work

b.

Fraction	Percent
$\frac{1}{5}$	20%
$\frac{2}{5}$	
$\frac{3}{5}$	
$\frac{4}{5}$	

c.

Fraction	Percent
$\frac{1}{25}$	4%
$\frac{2}{25}$	
$\frac{3}{25}$	
$\frac{4}{25}$	
$\frac{5}{25}$	
$\frac{6}{25}$	
$\frac{7}{25}$	

PERCENTS GREATER THAN 100%

What Lies Ahead

In this section you will learn these skills.

- interpreting percents greater than 100%

- expressing a percent as a decimal number and as a fraction

- expressing a fraction or decimal number as a percent

Working Together

In the previous section you worked with percents less than 100%. Now you will look at percents greater than 100%.

Notice how these percents can be written as whole numbers, mixed numbers, or decimal numbers.

Example 1

This container of popcorn represents 100% of the popcorn.

popcorn

$$100\% = \frac{100}{100} = 1$$

These containers represent 200% of the popcorn.

popcorn   popcorn

$$200\% = \frac{200}{100} = 2$$

These containers represent 250% of the popcorn.

popcorn   popcorn   popcorn
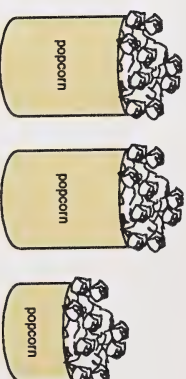
$$250\% = \frac{250}{100} = 2\frac{1}{2}$$

You already know that $2\frac{1}{2} = 2.5$.

Example 2

This rain barrel can represent 100% of the rain that fell last year.
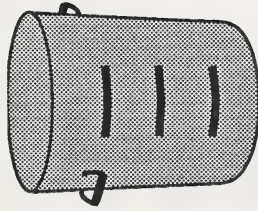
$$100\% = \frac{100}{100} = 1$$

The rainfall this year is 125% of the rainfall last year.

$$125\% = \frac{125}{100}$$
$$= 1\frac{25}{100}$$
$$= 1\frac{1}{4}$$

The rainfall that fell this year can be represented by these barrels.
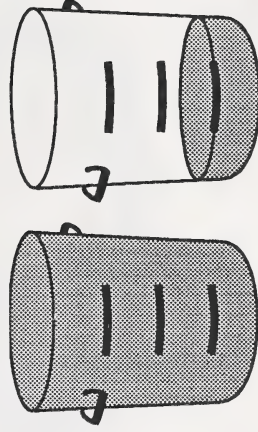
Introductory Activities

1. This year the hens laid 300% of the eggs they laid last year. Express the percent as a whole number.

2. Joan repaid her mother 110% of the money she borrowed.

   a. Express the percent as a fraction.

   b. Express the percent as a decimal number.

3. Becky is 125% of the height she was two years ago.

   a. Express the percent as a fraction.

   b. Express the percent as a decimal number.
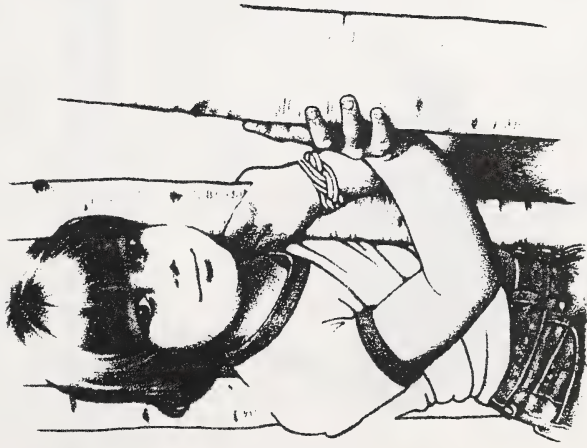
Working Together

Whole numbers and mixed numbers can be expressed as percents.

Example 1

A penguin egg hatches in 5 times the amount of time in which a sparrow egg hatches. Express this as a percent.

$$5 = \frac{5}{1}$$
$$= \frac{500}{100}$$
$$= 500\%$$

The time for a penguin egg to hatch is 500% of the time for a sparrow egg to hatch.

Example 2

An ostrich's mass is about $7\frac{1}{2}$ times that of the heaviest flying bird. Express this as a percent.

$$7\frac{1}{2} = \frac{15}{2}$$
$$= \frac{750}{100}$$
$$= 750\%$$

The mass of an ostrich is 750% of the mass of the heaviest flying bird.

Example 3

The speed at which the canvas-back duck can fly is 2.3 times the speed at which an ostrich can run. Express this as a percent.

$$2.3 = \frac{2.3}{1}$$

$$= \frac{230}{100}$$

$$= 230\%$$

The speed at which a canvas-back duck can fly is 230% of the speed at which an ostrich can run.
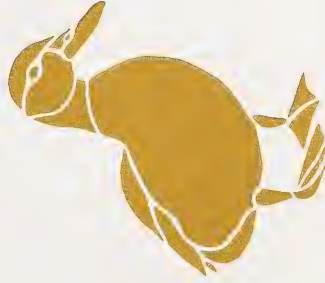
Example 4

Some jet planes travel at a speed that is about 6 times the speed of the spine-tailed swift.

Express this as a percent.

$$6 = \frac{6}{1}$$

$$= \frac{600}{100}$$

Some jet planes fly at 600% of the speed of the spine-tailed swift.

127
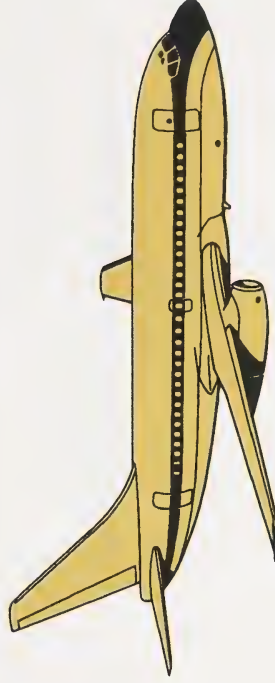
Practice Activities

Space for Your Work

1. a. A supersonic jet can fly at 2.2 times the speed of sound. Express this as a percent.

   b. A rocket can fly at $5\frac{3}{4}$ times the speed of sound. Express this as a percent.

2. a. A nail is $2\frac{1}{2}$ times the length of a paper clip. Express this as a percent.

   b. A pencil is 1.6 times the length of a nail. Express this as a percent.

Space for Your Work

3. The diameter of the wire to make coat hangers is
   $7\frac{1}{2}$ times the diameter of a human hair. Express this
   as a percent.

4. Milk is 1.56 times the mass of gasoline. Express this
   as a percent.

See your learning facilitator to check
your answers and to receive further
instructions.

Concluding Activities

1. An ostrich's egg is 1 000 000 times the mass of a hummingbird's egg. Express this as a percent.

2. The volume of steam is 167 000 000% of the volume of water. Express this as a whole number.

3. An ant can lift a mass that is 5000% of the mass of its own body. Express this as a whole number.
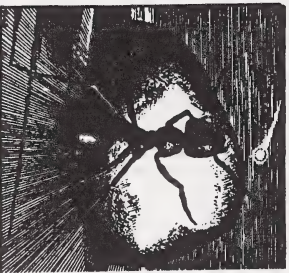
See your learning facilitator to check your answers and to receive further instructions.

PROPORTION AND PERCENT

What Lies Ahead

In this section you will learn to use this skill.

- Using proportions to solve problems of this form: What percent is *a* of *b*?

Working Together

As you have learned, percents can be less than 100% or greater than 100%. It depends on what is being compared.

Be very careful to use the correct order when expressing a ratio as a percent.

Example

There are 5 black marbles, 10 white marbles, and 10 brown marbles in this group of 25 marbles.
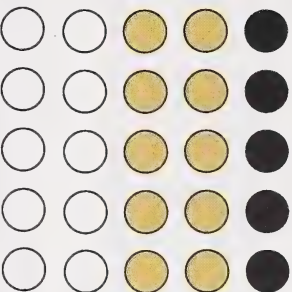
What percent of the total are black marbles?

Solution

Write the ratio of black marbles to the total.

$$\frac{5}{25}$$

To write the percent, find an equivalent ratio with a second term of 100.

$$\frac{5}{25} = \frac{\blacksquare}{100}$$

Find the missing term.

$$\frac{5}{25} = \frac{20}{100}$$
×4 ... ×4

The number of black marbles is 20% of the total.

The number of black marbles is what percent of the number of white marbles?

Solution

Write the ratio of black marbles to white marbles in simplest form.

$$\frac{5}{10} = \frac{1}{2}$$

To write the percent, find an equivalent ratio with a second term of 100.

$$\frac{1}{2} = \frac{\blacksquare}{100}$$

Find the missing term.

$$\overset{\times 50}{\overbrace{\frac{1}{2}}} = \underset{\times 50}{\underbrace{\frac{50}{100}}}$$

The number of black marbles is 50% of the number of white marbles.

The number of brown marbles is what percent of the number of black marbles?

Solution

Write the ratio of brown marbles to black marbles in simplest form.

$$\frac{10}{5} = \frac{2}{1}$$

To write the percent, find an equivalent ratio with a second term of 100.

$$\frac{2}{1} = \frac{\blacksquare}{100}$$

Find the missing term.

$$\overset{\times 100}{\overbrace{\frac{2}{1}}} = \underset{\times 100}{\underbrace{\frac{200}{100}}}$$

The number of brown marbles is 200% of the number of white marbles.

Introductory Activities

1. 2 out of 25 light-bulbs tested by the Bright Light Company failed. Express this as a percent.
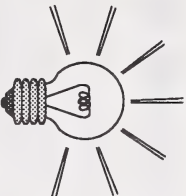
2. There are 23 boys and 20 girls on the bus. Express this ratio as a percent. Use a statement.

Space for Your Work
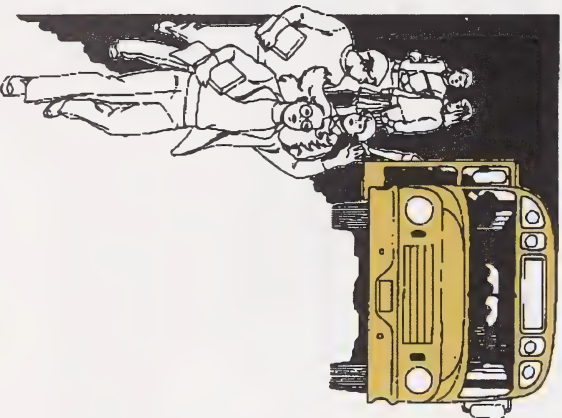
3. Janice received these marks on five tests.

Language Arts	19 out of 25
Mathematics	42 out of 50
Science	9 out of 10
Social Studies	16 out of 20
French	82 out of 100

a. Express each mark as a percent.

b. Rank the test results from the highest percent to the lowest percent.

See your learning facilitator to check your answers and to receive further instructions.

Working Together

You can use a calculator to help you find the percent.

Example 1

There are 24 boys and 15 girls in a group. What is the percent of boys to girls?

Solution

Write the ratio of boys to girls.

$$\frac{24}{15}$$

Use a calculator to find the decimal number form.

Key Press	Display
2 4 ÷ 1 5 =	1.6

$$\frac{24}{15} = 1.6$$
$$= 1.60$$
$$= 160\%$$

The number of boys is 160% of the number of girls.

Example 2

There are 0.3 g of dietary fibre in 30 g of breakfast cereal. What percent of the cereal is dietary fibre?

Solution

Write the ratio of dietary fibre to cereal.

$$\frac{0.3}{30}$$

Use a calculator to find the decimal number form.

Key Press	Display
0 . 3 ÷ 3 0 =	0.01

$$\frac{0.3}{30} = 0.01$$
$$= 1\%$$

The amount of dietary fibre is 1% of the cereal.

Practice Activities

1. Kerry has 200 customers on a paper route. 36 of the customers are away on holidays. What percent of the customers are away?

2. Howie got 36 questions correct on a 45-question test. What percent did he get correct?

3. John earns $30 000 a year. He spends $9000 on taxes. What percent of his salary does he spend on taxes?

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

1. Bronze is made of 9 parts copper and 1 part tin.

   a. What percent of the bronze is tin?

   b. What percent of the bronze is copper?

   c. The number of parts of copper is what percent of the number of parts of tin?

2. a. A punch has cranberry juice, lemonade, and ginger ale in the ratio of 1 : 3 : 2. What percent of the punch is lemonade?

   b. Cranberry juice is what percent of the ginger ale in the punch?

See your learning facilitator to check your answers and to receive further instructions.

Space for Your Work

FINDING A PERCENT OF A NUMBER

What Lies Ahead

In this section you will learn these skills.

* estimating a percent of a number

* finding a percent of a number using paper and pencil methods or a calculator

* calculating a percent of a number using mental computation

Working Together

Percents are very useful when describing situations. Unfortunately, percents cannot be used in calculations. To do calculations, you must change percents to fractions or decimal numbers.

Estimating

Sometimes you only need to estimate the percent of a number in everyday life.

Example 1

Mr. Allan decides to leave his waiter 15% of the food bill as a tip. If the food bill was $67.98, how much should Mr. Allan leave as a tip?

Solution

An estimate is sufficient in this situation. Remember that estimation is done mentally.

Method 1

To make an estimate, you can use rounding.

15% of 67.98 ≐ 15% of 70
= 0.15 × 70
≐ 70 × 0.15
≐ 10.50

Mr. Allan's tip for the waiter should be about $10.50.

Method 2

You can also use front-end digits to make an estimate.

15% of 67.98 ≐ 10% of 60
= 0.1 × 60
= 6.00

Front-end digits will give a low estimate, so you should compensate for the difference.

5% of 60 = 0.05 × 60
= 3.00
So, 15% of 67.98 ≐ 6 + 3
≐ 9

Mr. Allan's tip for the waiter should be about $9.

At other times you need to find an exact answer.

Example 2

Mr. Kurtz sells cars. He earns 5% of his sales. If Mr. Kurtz sells a car for $18 050, how much does he earn on this one car alone?
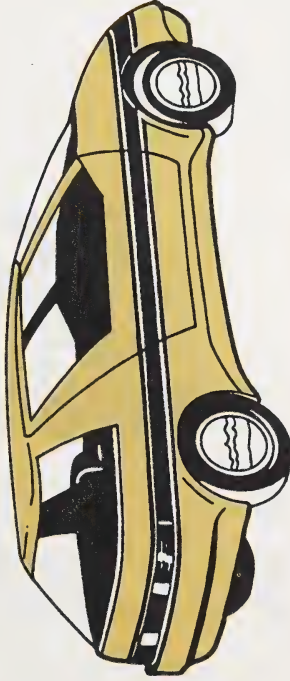
Solution

An exact answer is not required here.

Even if an exact answer is required, an estimate will provide a "ballpark answer" with which to compare the calculated answer.

Method 1

To make an estimate, you can use rounding.

$$5\% \text{ of } 18\,050 \doteq 5\% \text{ of } 20\,000$$
$$\doteq 0.05 \times 20\,000$$
$$\doteq 1000$$

Mr. Kurtz would earn about $1000 on this car alone.

Method 2

You can also use front-end digits to make an estimate.

$$5\% \text{ of } 18\,050 \doteq 5\% \text{ of } 10\,000$$
$$\doteq 0.05 \times 10\,000$$
$$\doteq 500$$

This is a low estimate, so you will need to compensate for the difference.

$$5\% \text{ of } 8\,000 = 0.05 \times 8000$$
$$= 400$$
$$\text{So, } 5\% \text{ of } 18\,050 \doteq 500 + 400$$
$$\doteq 900$$

Mr. Kurtz would earn about $900 on this car alone.

When you are estimating using rounding, you may round either some of the numbers or all of the numbers.

Example 1

A package of cheese has 23% milk fat. If the package holds 275 g of cheese, about how much milk fat is there in the package?

Solution

The amount of milk fat is estimated like this.

"of" means multiply

↓

23% of 275 = 0.23 × 275
$\doteq$ 0.2 × 300
$\doteq$ 60

There are about 60 g of milk fat in 275 g of cheese.

Example 2

Andy scored 73% on a test. If there were 35 questions on the test, about how many questions did Andy get right?

Solution

The number of questions Andy got right is estimated like this.

"of" means multiply

↓

73% of 35 = 0.73 × 35
$\doteq$ 0.7 × 35
$\doteq$ 24.5

Andy got about 25 questions right out of a total of 35 questions.

Introductory Activities

Space for Your Work

For Questions 1 to 5, circle the best estimate.

1. 98% of 680
   a. 0.68
   b. 6.8
   c. 68
   d. 680

2. 26% of 399
   a. 1
   b. 10
   c. 100
   d. 1000

3. 33% of 180
   a. 0.6
   b. 6
   c. 60
   d. 600

4. 49% of 105
   a. 0.5
   b. 5
   c. 50
   d. 500

5. 22% of 450
   a. 0.9
   b. 9
   c. 90
   d. 900

6. Estimate an answer for each of the following.

a. 98% of 60

b. 11% of 750

c. 22% of 50

d. 20% of 189

e. 50% of 59

f. 82% of 605

Computer Alternative

7. Do the program "Making Sense of Percents" on Disk B of *MAC 6*. Read the instructions in the folder with the disk before using the program.

See your learning facilitator to check your answers and to receive further instructions.

Space for Your Work

Working Together

Calculating

The volleyball team sold $375 worth of chocolate bars. If they kept 20% of the sales, how much did they keep for themselves?
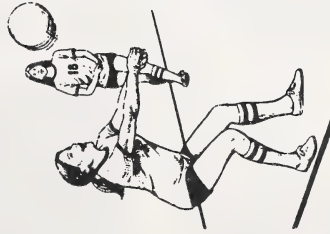
Solution

Method 1

You can calculate using paper and pencil.

20% of 375 = 0.20 × 375

= 75

The team kept $75 for themselves.

Method 2

You can use a calculator to find a whole number value for the percent of a number.

20% of 375 = 0.20 × 375

Key Press	Display
0 . 2 × 3 7 5 =	75

Most calculators have a % key.

This saves the need of changing the percent to a decimal number and the need for pressing the = key. This is what happens in this case.

Key Press	Display
3 7 5 × 2 0 %	75

Note

Enter the number to be multiplied first and the percent key last.

Practice Activities

Space for Your Work

1. The seating capacity of a football stadium is 10 800. At one game 75% of the seats were occupied. How many people were at the football game?

2. About 80% of the nuts in a can of mixed nuts are peanuts. In a handful of 20 nuts, how many peanuts are there?

Space for Your Work

3. Find the amount of milk fat contained in 1000 mL of each type of dairy product in the chart below.

Type	Milk Fat
a. whipping cream	35%
b. coffee cream	18%
c. cereal cream	9%
d. partly-skimmed milk	2%
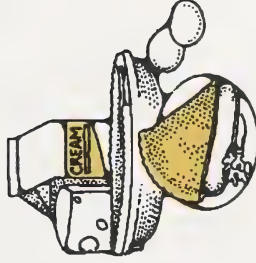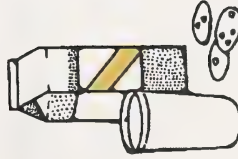
4. Martinez uses solder to fix his pipes. Solder is a mixture of tin and lead. In hard solder, 65% of the total mass is tin. Find the amount of lead and tin in a 5-kg bar of hard solder.
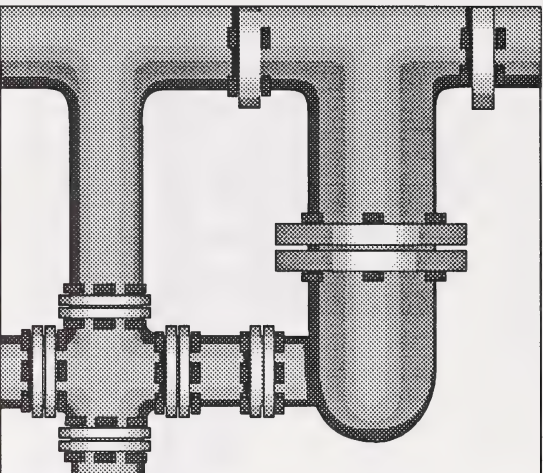
Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Working Together

When you find a percent of a number, you may either use the decimal number form or the fraction form of the percent.

Sometimes it is easier to work with the decimal number form.

Example 1

What is 10% of 31.20?

Think   10% of 31.20 = 0.1 × 31.20

= 3.12

$$
\begin{array}{rl}
31.20 & \rightarrow \text{2 decimal places} \\
\times\ 0.1 & \rightarrow \text{1 decimal place} \\
\hline
3.12 & \rightarrow \text{3 decimal places}
\end{array}
$$

Example 2

What is 20% of 31.20?

Think   10% of 31.20 = 0.1 × 31.20

= 3.12

20% of 31.20 = 6.24

Find 10% and double this amount.

Example 3

What is 30% of 31.20?

Think   10% of 31.20 = 3.12

30% of 31.20 = 9.36

Find 10% and triple this amount.

At other times it is easier to work with the fraction form of the percent.

Example 1

What is 20% of 50?

Think

$$20\% \text{ of } 50 = \frac{1}{5} \times 50$$

$$= 10$$

> Multiplying by $\frac{1}{5}$ is the same as dividing by 5.

Example 2

What is 80% of 50?

Think

$$20\% \text{ of } 50 = \frac{1}{5} \times 50$$

$$= 10$$

$$80\% \text{ of } 50 = \frac{4}{5} \times 50$$

$$= 40$$

> Find 20% and multiplying by 4.

Example 3

What is 25% of 40?

Think

$$25\% \text{ of } 40 = \frac{1}{4} \times 40$$

$$= 10$$

Example 4

What is 75% of 40?

Think

$$25\% \text{ of } 40 = \frac{1}{4} \times 40$$

$$= 10$$

$$75\% \text{ of } 40 = \frac{3}{4} \times 40$$
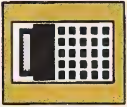
$$= 30$$

> Find 25% and multiply by 3.

Concluding Activities

1. Complete each of the following mentally.

   a. 25% of 16

   b. 50% of 16

   c. 75% of 16

   d. 10% of 90

   e. 20% of 90

   f. 30% of 90

2. Compute each of the following using a calculator.

   a. 18% of 50

   b. 50% of 18

   c. 88% of 25

   d. 25% of 88

3. What pattern did you notice in Question 2?

4. Apply the pattern you discovered in Question 3 to compute each of the following mentally.

   a. 26% of 50

   b. 84% of 25

   c. 55% of 20

Space for Your Work

5. Compute each of the following using a calculator.

    a.   45% of 60

    b.   20% of 60 + 25% of 60

    c.   60% of 42

    d.   50% of 42 + 10% of 42

    e.   19% of 24

    f.   20% of 24 – 1% of 24

6.   What pattern did you notice in Question 5?

7.   Apply the pattern you discovered in Question 6 to compute each of the following mentally.

    a.   35% of 80

    b.   31% of 60

    c.   79% of 50

    d.   11% of 40

Space for Your Work

8. Compute each of the following using a calculator.

   a. 25% of 48

   b. 50% of 24

   c. 15% of 70

   d. 30% of 35

   e. 90% of 50

   f. 45% of 100

9. What pattern did you discover in Question 8?

10. Apply the pattern you discovered in Question 8 to calculate each of the following mentally.

    a. 15% of 32

    b. 50% of 36

    c. 22% of 50

    d. 36% of 200

See your learning facilitator to check your answers and to receive further instructions.

Notes

Notes

APPLICATION OF PERCENT
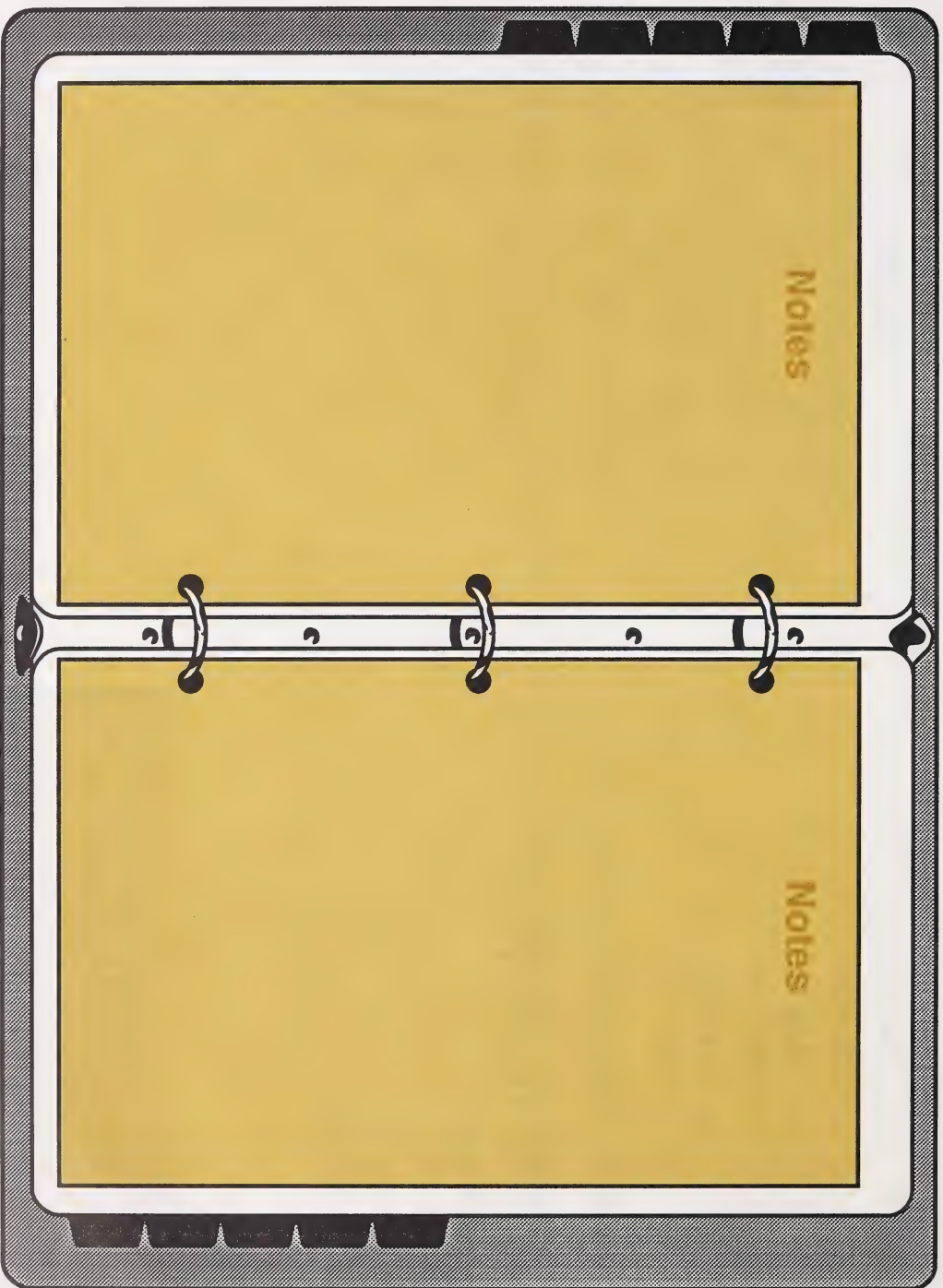
What Lies Ahead

In this section you will learn these skills.

* finding percent increases and decreases

In this section you will use these words.

* discount
* sales tax
* regular price
* sale price

Working Together

One of the most common applications of percent can be found when you go shopping. It is very useful if you are able to mentally calculate discounts and sales taxes so that you can figure out the price that you pay.

Discount

Almost every day you encounter the term **discount**.
Discount can be defined as the amount by which the
regular price is reduced. What does this mean in dollars
and cents?

Example 1

A store is selling video games at a 5% discount. If a
video game is regularly priced at $40, what is its price
with the 5% discount?

A 5% discount means that the store is willing to take 5%
of the regular price and subtract it from the regular price.
The price that you now pay is called the **sale price**.

$$\text{Sale price} = \text{Regular price} - \text{Discount}$$

Mathematically this can be calculated as follows.

$$
\begin{aligned}
\text{Discount} &= 5\% \text{ of the regular price} \\
&= 5\% \text{ of } \$40 \\
&= 0.05 \times 40 \\
&= 2
\end{aligned}
$$

$$5\% = \frac{5}{100}$$
$$= 0.05$$

The $2 is called the discount. This is the amount of
money the store is willing to take off the regular price.

$$
\begin{aligned}
\text{Sale price} &= \text{Regular price} - \text{Discount} \\
&= 40 - 2 \\
&= 38
\end{aligned}
$$

The sale price for this video game is $38.

Example 2

What is the value of the discount on a pair of shoes that are regularly priced at $179.99 but that are now selling at a 25% discount?

Solution

Make an estimate first.

25% of $179.99 $\doteq$ 25% of $200

$$\doteq \frac{1}{4} \times 200$$

$$\doteq 50$$

Then calculate the exact value.

25% of 179.99 = 0.25 × 179.99

$$= 44.9975$$

$$= 45.00$$

The discount is $45.

Note

Store prices are always rounded up.

$$25\% = \frac{1}{4}$$
$$179.99 \doteq 200$$

Sales Tax

In Canada a **goods and services tax (GST)** of 7% is added on to the price of most items you buy.

In all provinces except Alberta, a provincial sales tax (PST) is also charged on these items. The provincial sales taxes vary.

Province	Sales Tax
Newfoundland	12%
New Brunswick	11%
Nova Scotia	10%
Prince Edward Island	10%
Quebec	9%
Ontario	8%
Manitoba	7%
Saskatchewan	7%
British Columbia	6%
Alberta	0%

Example

What would a pair of skis cost a customer in Alberta, Newfoundland, and British Columbia if the selling price is $418.98?

Solution

Alberta

In Alberta there is no provincial sales tax.

Only the goods and services tax (GST) is calculated.

7% of $418.98 = 0.07 × $418.98

$$= \$29.3286$$

$$\doteq \$29.33$$

Cost = Selling price + GST

$$= \$418.98 + \$29.33$$

$$= \$448.31$$

So, the cost of the skis to a customer in Alberta is $448.31.

Newfoundland

In the Altantic provinces and Quebec, the GST is calculated first and added to the selling price.

This is the same as the cost in Alberta, or $448.31.

Then the provincial sales tax (PST) is calculated.

12% of $448.21 = 0.12 × $448.31

$$= \$53.7972$$

$$\doteq \$53.80$$

Final cost = (Selling price + GST) + PST

$$= \$448.31 + \$53.80$$

$$= \$502.11$$

So, the cost of the skis to a customer in Newfoundland is $502.11.

British Columbia

In the provinces other than the Atlantic provinces, the GST and PST are calculated side by side.

(7% + 6%) of 418.98 = 13% of $418.98

$$= 0.13 × 418.98$$

$$= \$54.4674$$

$$= \$54.47$$

Cost = Selling price + (GST + PST)

$$= \$418.98 + 54.47$$

$$= \$473.45$$

So, the cost of the skis to a customer in British Columbia is $473.45.

Practice Exercises

1. What discount is given on each of the following items?

   a. A CD player regularly costs $159.99. It is on sale at a discount of 18%.

   b. A 10-speed bicycle is on sale at a discount of 25%. It regularly costs $250.

   c. Hockey gloves that regularly cost $39 are on sale at a discount of 40%.

2. Calculate the sale price for each of the following items.

   a. A TV that regularly costs $950 is on sale at a discount of 25%.

   b. A microwave oven that regularly costs $249.99 is on sale at a discount of 20% off.

3. What is GST and sales tax for each of the following items?

   a. A skateboard costs $89.99 in British Columbia.

   b. A watch costs $17.50 in Saskatchewan.

   c. Running shoes cost $49.95 in Nova Scotia.

See your learning facilitator to check your answers and to receive further instructions.

Working Together

When solving problems that involve discount or sales tax, alternative methods can be used.

Example 1

An item that has a regular price of $60 is on sale for 30% off. What is the sale price of this item?

Solution

30% off means that you pay 70% of the regular price.

$$100\% - 30\% = 70\%$$

| Regular price | – | Discount | = | Final cost |
| 100% | | 30% | | 70% |

To find the sale price, calculate the value of 70% of the regular price.

$$70\% \text{ of } \$60 = 0.70 \times \$60$$
$$= \$42$$

The sale price of the item is $42.
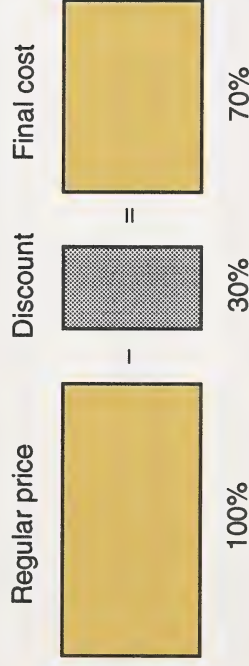
Example 2

The GST is 7%. Find the total cost of an item that sells for $18 in Alberta.

Solution

An extra charge of 7% means that you pay 107% of the regular price.

$$100\% + 7\% = 107\%$$

| Regular price | + | GST | = | Final cost |

100%   7%   107%

$$
\begin{aligned}
\text{Final cost} &= (100\% + 7\%) \text{ of regular price} \\
&= 107\% \text{ of } 18 \\
&= 1.07 \times 18 \\
&= 19.26
\end{aligned}
$$

The final cost of the item would be $19.26.
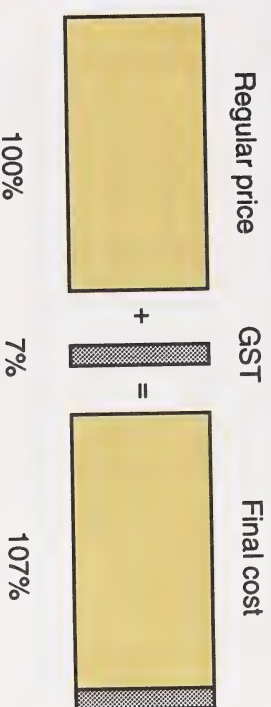
Extra Practice

1. A department store is offering a 30% discount on regularly priced items. Find the sale price of each of these items.

   a. a coat that is regularly priced at $35

   b. shoes that are regularly priced at $89.99

   c. a dress that is regularly priced at $150

   d. a suit that is regularly priced at $249.95

2. A car is priced at $18 500 in Alberta and the GST is 7%. What is the final cost of the car?

3. A sweater sells for $39 in Manitoba, the GST is 7%, and the sales tax is 7%. What is the final cost?

See your learning facilitator to check your answers and to receive further instructions.

Space for Your Work

Concluding Activities

1. A sporting goods store is having a 20% off sale. Calculate the final cost of each of the following items. Use your calculator.

   a. a basketball that is regularly priced at $39.99

   b. a hockey stick that regularly costs $28.98

2. If the GST is 7%, find the final cost of each purchase in Alberta. Use your calculator.

   a. jeans that cost $55

   b. boots that cost $89.99

   c. a 10-speed bicycle that costs $259

   d. a tape that costs $12.98

See your learning facilitator to check your answers and to receive further instructions.

Space for Your Work

![icon] **Working Together**

Using the Calculator to Subtract Discounts

A calculator can save a great deal of time when subtracting discounts.

Example 1

A store is offering a discount on some items in the store. Calculate the sale price for the items regularly priced at $3.99 if they are marked 25% off.

Solution

There are two ways to solve this problem.

Method 1

Think

$$3.99 - 25\% \text{ of } 3.99 = 3.99 - 0.25 \times 3.99$$

If your calculator does not follow the rules for order of operations, you will have to use the memory keys.

Key Press	Display
0 . 2 5 × 3 . 9 9 M+	M **0.9975**
3 . 9 9 − MR =	M **2.9925**

The display indicates the sale price. $2.9925 rounded up becomes $3.00. So, the sale price is $3.00.

Method 2

Think

$$3.99 - 25\% \text{ of } 3.99 = 75\% \text{ of } 3.99 = 0.75 \times 3.99$$

Key Press	Display
0 . 7 5 × 3 . 9 9 =	**2.9925**

The display inidcates the sale price. $2.9925 rounded up is $3.00. So, the sale price is $3.00.

Note

The second method is easier. It does not require the use of the memory keys.

Example 2

A store is holding a 20% off sale on all men's and women's moccasins and women's flat shoes. Calculate the sale price for items regularly priced at $5.59, $6.89, and $7.38.

PHOTO SEARCH LTD.

Solution

Because 20% is repeatedly being taken off, it is simpler to find 80% of each item. You can use the memory keys to calculate the sale prices.

Key Press	Display
0 . 8 0 M+	M 0.80
5 . 5 9 X MR =	M 4.472
6 . 8 9 X MR =	M 5.512
7 . 3 8 X MR =	M 5.904

The sale price of items regularly priced at $5.59 is $4.472 or $4.48.

The sale price of items regularly priced at $6.89 is $5.512 or $5.52.

The sale price of items regularly priced at $7.38 is $5.904 or $5.91.

Using the Calculator to Add on Tax

A calculator can save a great deal of time when adding on tax.

Example 1

A store in Alberta calculates a goods and services tax and adds this to the cost of purchases. If you bought goods totalling $39.49 at this store, what would the cost be after adding on the taxes?

Solution

There are two ways to solve this problem.

Method 1

Think

$$39.49 + 7\% \text{ of } 39.49 = 39.49 + 0.7 \times 39.49$$

Key Press	Display
0 . 0 7 × 3 9 . 4 9 = M+	M 2.7643
3 9 . 4 9 + M+ MR =	M 42.2543

The price after adding tax is $42.26.

Method 2

Think

$$39.49 + 7\% \text{ of } 39.49 = 107\% \text{ of } 39.49$$
$$= 1.07 \times 39.49$$

Key Press	Display
1 . 0 7 × 3 9 . 4 9 =	42.2543

The price after adding tax is $42.26.

Note

The second method is easier. It does not require the use of the memory keys.

If your calculator does not follow the rules for order of operations, you will have to use the memory keys.

Example 2

A bakery in Alberta adds the goods and services tax of 7% onto the purchase price. What is the final cost of purchases totaling $15.85 and $7.24?
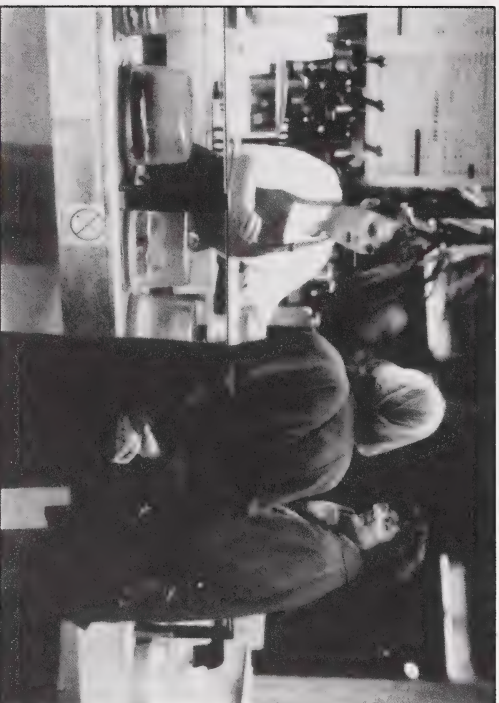

PHOTO SEARCH LTD.

Solution

Because 7% is repeatedly being added on, it is simpler to find 107% of each total. You can use the memory keys of the calculator to find the prices after tax.

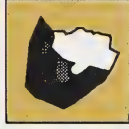| Key Press	Display
$\boxed{1}\boxed{.}\boxed{0}\boxed{7}\boxed{M+}$	M **1.07**
$\boxed{1}\boxed{5}\boxed{.}\boxed{8}\boxed{5}\boxed{\times}\boxed{MR}\boxed{=}$	M **16.9595**
$\boxed{7}\boxed{.}\boxed{2}\boxed{4}\boxed{\times}\boxed{MR}\boxed{=}$	M **7.7468**

The price of the first purchases after tax is $16.96.

The price of the second purchases after tax is $7.75.

SUMMARY

What Lies Ahead

In this section you will review the skills you have learned in Sections 11 to 16.

Working Together

At this time it is a good idea to review Part Three.

Video Activity

You may wish to watch the video *Solve It: The Meaning of Percent* as a review.

Turn to Section 11 in this Module Booklet and correct any errors you may have made in the pretest. You may be pleasantly surprised to discover how much you have learned about percents!

What Lies Ahead

In this section you will complete the Module Assignment.

Working Together

Now that you have completed Module 5 and you have done the required practice, you should be ready for the Module Assignment.

Module Assignment

Turn to the Assignment Booklet and complete the Module Assignment independently. You may refer to your notes, but do not get help from anyone.

APPENDIX

GLOSSARY

Discount: the amount by which a regular price is reduced

Equivalent ratios: ratios that have the same simplest form

$\frac{4}{10}$ and $\frac{6}{15}$ are equivalent ratios.

First term: the first number in a ratio

2 is the first term of 2 : 3.

Gear ratio: the ratio of the number of teeth on the first gear to the number of teeth on the second gear

Golden ratio: the ratio of the length to width of the most eye-appealing rectangles

This ratio is 1.618.

Goods and services tax: an extra charge added on to the price of some goods and services

Lowest-term ratio: a ratio using the smallest whole numbers possible

Percent: a ratio that compares an amount to 100

85% means $\frac{85}{100}$.

Proportion: an equation showing that two ratios are equal

$\frac{4}{5} = \frac{24}{30}$ is a proportion.

Proportional: having the same ratio

Proportional ratios: ratios that are equivalent

$\frac{2}{3}$ and $\frac{4}{6}$ are proportional ratios.

Rates: special ratios that compare quantities with different units

Ratio: a comparison of two numbers

Sales tax: an extra charge added on to the price

Second term: the second number of a ratio

3 is the second term of 2 : 3.

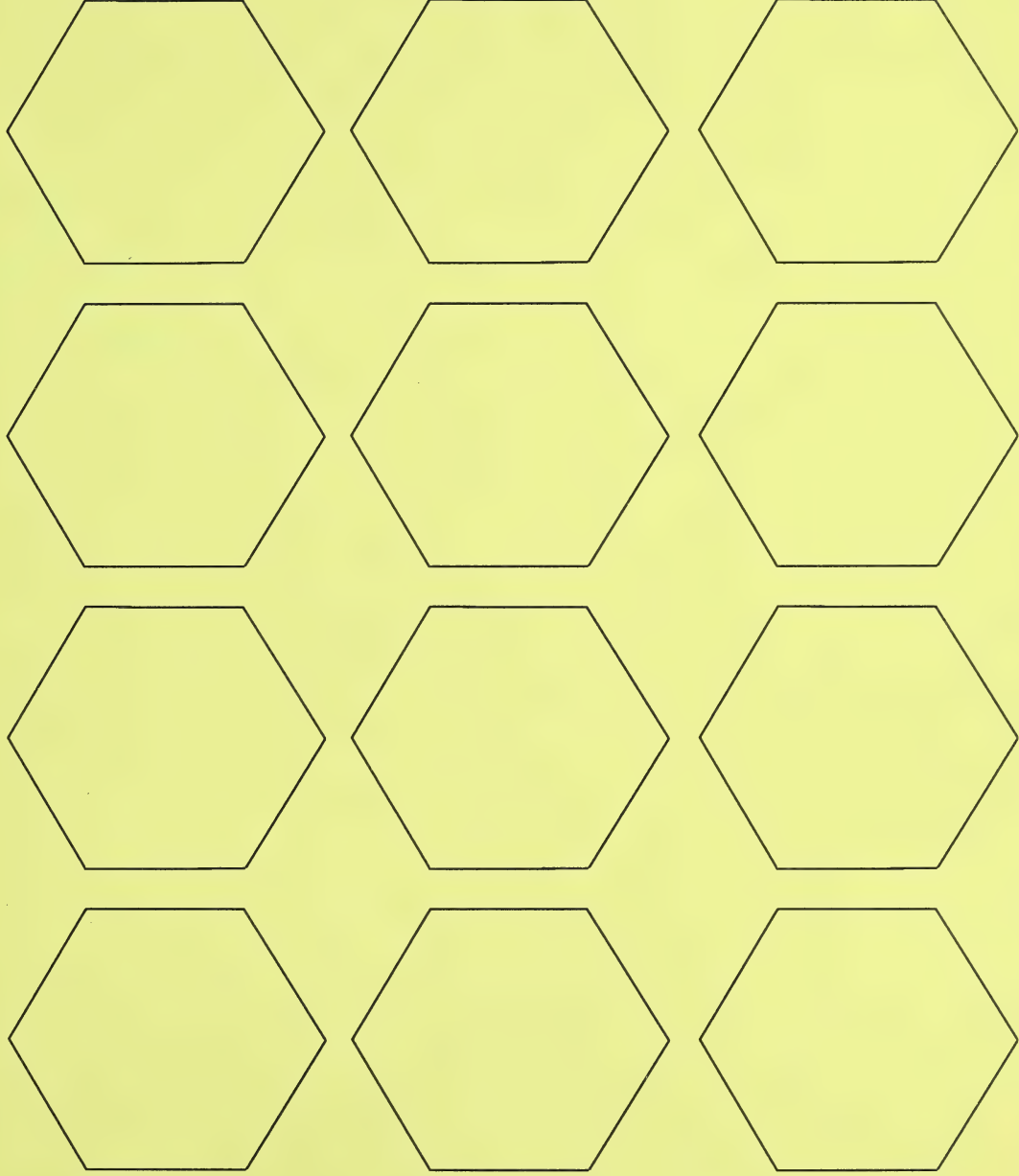**Term:** the numbers in a ratio

$\dfrac{2}{3}$ ← first term
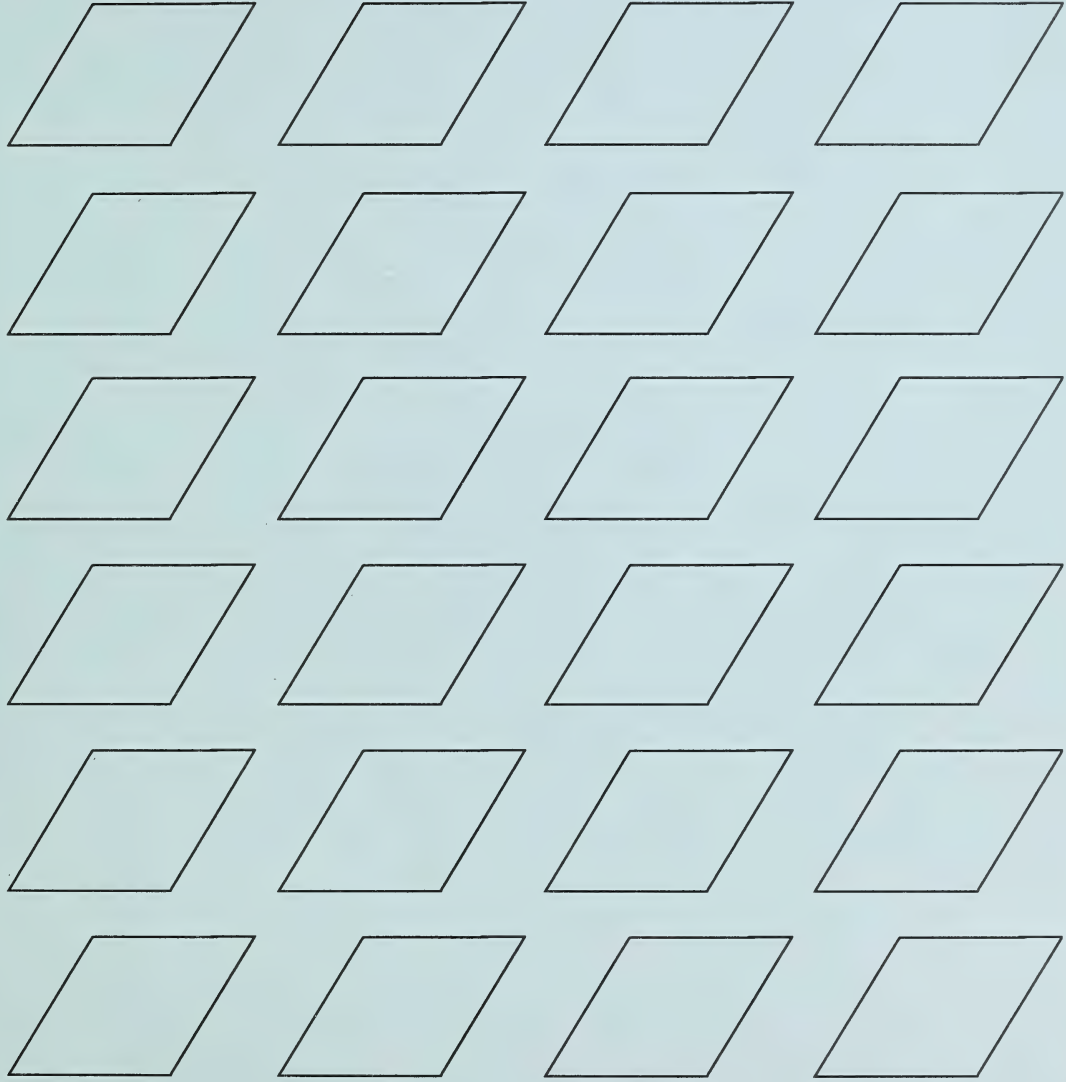← second term

Third term: the third number of a ratio

5 is the third term of 2 : 3 : 5.

Three-term ratios: ratios that have three terms

Two-term ratios: ratios that have two terms

HEXAGONS

PARALLELOGRAMS

TRIANGLES